## Math 22: Unit 3 Exam

## Spring Semester 2006

**Instructions.** Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. For any short answer questions, write clearly your answer and any additional explanation that is needed.

- 1. (10 points) Use the limit comparison test to determine whether or not  $\sum_{n=1}^{\infty} \frac{3n^3}{n^4 + n^3 + 4}$  converges or diverges.
- 2. (10 points) What is the radius of convergence for  $\sum_{n=0}^{\infty} \frac{n}{5^n} (2x-1)^n$ ?
- 3. (5 points) Suppose a function f satisfies f(2) = 4, f'(2) = 3, f''(2) = -5, and f'''(2) = 12. What is the third degree Taylor polynomial for f about x = 2?
- 4. (5 points) Use the integral test to determine whether or not the series  $\sum_{n=1}^{\infty} \frac{2n}{(n^2+1)^3}$  converges.
- 5. For the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n^2}$ ,
  - (a) (5 points) Use the alternating series test to determine whether or not this series converges.
  - (b) (5 points) Is this series absolutely convergent? Explain why or why not.
- 6. (5 points) Find the exact value for  $\sum_{n=0}^{\infty} \frac{2+4^n}{5^n}$ .
- 7. (5 points) Use the Taylor series of  $\cos x$  about x = 0 to determine  $\lim_{x \to 0} \frac{1 \cos x}{x^2}$ .