For Problems #1-#2, evaluate the integral.

1. \[
\int_{1}^{e} t \ln t \, dt
\]
   Solution: Use integration by parts with \( u = \ln t \) and \( dv = t \, dt \), so \( du = \frac{1}{t} \, dt \) and \( v = \frac{t^2}{2} \, dt \). Using this substitution, we get
   \[
   \int_{1}^{e} t \ln t \, dt = \int_{1}^{e} \frac{t^2}{2} \, dt = \left. \frac{t^2}{4} \right|_{1}^{e} = \frac{e^2 - 1}{4}
   \]

2. \[
\int_{1}^{2} \frac{dx}{\sqrt{2x-x^2}}
\]
   Solution: The expression \( 2x - x^2 \) can be written as \( -(x^2 - 2x) \). We complete the square on \( x^2 - 2x \), getting \( (x - 1)^2 - 1 \). Hence the integral can be written as
   \[
   \int_{1}^{3} \frac{dx}{\sqrt{1 - (x-1)^2}}
   \]
   Now use the substitution \( u = x - 1 \), \( du = dx \), and change the limits accordingly, to get
   \[
   \int_{0}^{1} \frac{du}{\sqrt{1 - u^2}}
   \]
   This expression is simply \( \sin^{-1} u \big|_{0}^{1} = \frac{\pi}{2} \).

3. Find the partial fraction decomposition of the following function. You do not need to integrate.

\[
\frac{x - 3}{x^2 - 4x + 4}
\]
   Solution: The denominator has been factored as \( x^2 - 4x + 4 = (x - 2)^2 \). Being a multiple factor, the appropriate form for the partial fraction decomposition is
   \[
   \frac{x - 3}{(x - 2)^2} = \frac{A}{x - 2} + \frac{B}{(x - 2)^2}
   \]
Multiplying both sides of the above equation by \((x - 2)^2\), we obtain

\[ x - 3 = A(x - 2) + B \]

This is true for any \(x\), so we can take \(x = 2\) to get \(-1 = B\). Then we may take \(x = 3\) (for instance; any choice will do) to get \(0 = A + B = A + -1\). Thus \(A = 1\). It follows then that the solution is

\[ \frac{x - 3}{(x - 2)^2} = \frac{1}{x - 2} - \frac{1}{(x - 2)^2} \]

4. Does the following integral converge or diverge. If it converges, find its value.

\[ \int_1^\infty \frac{1}{x(ln x)^3} \, dx \]

Solution: We write the improper integral as a limit:

\[ \int_1^\infty \frac{1}{x(ln x)^3} \, dx = \lim_{t \to +\infty} \int_1^t \frac{1}{x(ln x)^3} \, dx \]

If we use the substitution \(u = \ln x, \, du = dx/x\), the proper integral becomes

\[ \int_1^{\ln t} \frac{1}{u^3} \, du \]

By FTC, we then obtain

\[ -\frac{1}{2u^2} \bigg|_1^{\ln t} = -\frac{1}{2} \cdot \frac{1}{(\ln t)^2} \]

Taking the limit as \(t \to +\infty\), we get \(\frac{1}{2}\). Thus the integral converges to \(\frac{1}{2}\).