SOLUTIONS FOR MATH 22 EXAM 3

1. Express the arclength of the curve $y = e^x$, $0 \le x \le 1$ as an integral. You do not need to evaluate the integral.

   Solution: The general formula for the arclength of a graph with equation $y = f(x)$ with x-range $a \le x \le b$ is given by

   $$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

   So in this case, our solution is

   $$\int_0^1 \sqrt{1 + e^{2x}} dx.$$

2. Find the area of the surface generated by rotating the curve in Problem #1 around the x-axis.

   Solution: The general formula for the area of the surface resulting from rotating the curve $y = f(x)$, $a \le x \le b$ around the x-axis is given by

   $$\int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

   In our case, with $y = e^x$, $0 \le x \le 1$, this becomes

   $$\int_0^1 2\pi e^x \sqrt{1 + e^{2x}} dx.$$

   I think that the best choice here is the substitution $u = e^x$, $du = e^x dx$, so that the integral becomes (changing the limits to $1 = e^0$ and $e = e^1$, of course)

   $$\int_1^e 2\pi \sqrt{1 + u^2} du$$

   Using the formula provided for you on the test, this is equivalent to

   $$\frac{1}{2} u \sqrt{1 + u^2} + \frac{1}{2} \ln(u + \sqrt{1 + u^2}) \Big|_1^e$$

   Plugging in the numbers, we get

   $$\frac{1}{2} e \sqrt{1 + e^2} + \frac{1}{2} \ln(e + \sqrt{1 + e^2}) - \frac{1}{2}\sqrt{2} - \frac{1}{2} \ln(1 + \sqrt{2})$$

   as our solution.

3. Let $\mathcal{C}$ be the curve traced out by the parametric equations

$$x = t^3 + t,$$
$$y = t^3 + t^2 + t.$$

a) Find all the points on $\mathcal{C}$ where the tangent line has slope 1.

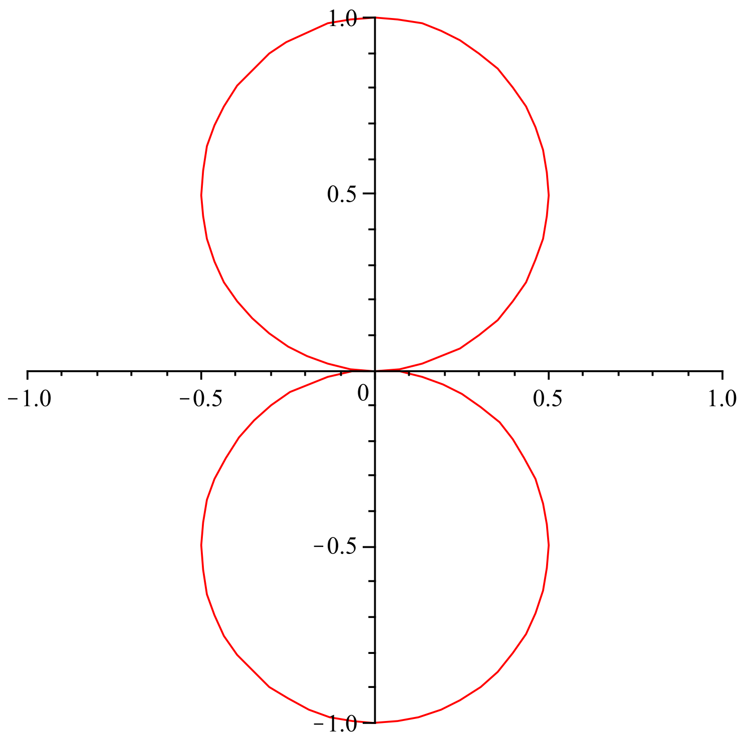
Solution: The tangent line at t is given by the formula $\frac{dy/dt}{dx/dt}$. In this case, it becomes $\frac{3t^2+2t+1}{3t^2+1}$. When this is set equal to one, we have $3t^2 + 2t + 1 = 3t^2 + 1$. This gives us $t = 0$, so the answer is $(0, 0)$.

b) At how many points does $\mathcal{C}$ intersect the line $y = x$?

Solution: Filling in the formulas in t in the equation $y = x$, we get $t^3 + t = t^3 + t^2 + t$. This gives $t^2 = 0$ or $t = 0$. So $(0, 0)$ is the answer.

4. Sketch the curve given by the polar equation $r = |sin\theta|$.

Solution: The graph looks like this:



5. There were lots of interesting correct explanations as to why c) was the right answer. For me, the fact that the curve couldn't get past -1.5 and 1.5 is an indication that there is a horizontal asymptote for y. Choice c) was the only parametic system that had a y function with horizontal asymptotes.