1. Express the arclength of the curve \( y = e^x \), \( 0 \leq x \leq 1 \) as an integral. You do not need to evaluate the integral.

Solution: The general formula for the arclength of a graph with equation \( y = f(x) \) with \( x \)-range \( a \leq x \leq b \) is given by

\[
\int_a^b \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx.
\]

So in this case, our solution is

\[
\int_0^1 \sqrt{1 + e^{2x}} \, dx.
\]

2. Find the area of the surface generated by rotating the curve in Problem #1 around the \( x \)-axis.

Solution: The general formula for the area of the surface resulting from rotating the curve \( y = f(x) \), \( a \leq x \leq b \) around the \( x \)-axis is given by

\[
\int_a^b 2\pi y \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx.
\]

In our case, with \( y = e^x \), \( 0 \leq x \leq 1 \), this becomes

\[
\int_0^1 2\pi e^x \sqrt{1 + e^{2x}} \, dx.
\]

I think that the best choice here is the substitution \( u = e^x \), \( du = e^x \, dx \), so that the integral becomes (changing the limits to \( 1 = e^0 \) and \( e = e^1 \), of course)

\[
\int_1^e 2\pi \sqrt{1 + u^2} \, du.
\]

Using the formula provided for you on the test, this is equivalent to

\[
\frac{1}{2} u \sqrt{1 + u^2} + \frac{1}{2} \ln(u + \sqrt{1 + u^2}) \bigg|_1^e
\]

Plugging in the numbers, we get

\[
\frac{1}{2} e \sqrt{1 + e^2} + \frac{1}{2} \ln(e + \sqrt{1 + e^2}) - \frac{1}{2} \sqrt{2} - \frac{1}{2} \ln(1 + \sqrt{2})
\]

as our solution.
3. Let \( C \) be the curve traced out by the parametric equations

\[
\begin{align*}
x &= t^3 + t, \\
y &= t^3 + t^2 + t.
\end{align*}
\]

a) Find all the points on \( C \) where the tangent line has slope 1.

Solution: The tangent line at \( t \) is given by the formula \( \frac{dy}{dt} = \frac{dy}{dx} \). In this case, it becomes \( \frac{3t^2 + 2t + 1}{3t^2 + 1} \). When this is set equal to one, we have \( 3t^2 + 2t + 1 = 3t^2 + 1 \). This gives us \( t = 0 \), so the answer is \((0, 0)\).

b) At how many points does \( C \) intersect the line \( y = x \)?

Solution: Filling in the formulas in \( t \) in the equation \( y = x \), we get \( t^3 + t = t^3 + t^2 + t \). This gives \( t^2 = 0 \) or \( t = 0 \). So \((0, 0)\) is the answer.

4. Sketch the curve given by the polar equation \( r = |\sin \theta| \).

Solution: The graph looks like this:
5. There were lots of interesting correct explanations as to why c) was the right answer. For me, the fact that the curve couldn’t get past -1.5 and 1.5 is an indication that there is a horizontal asymptote for \( y \). Choice c) was the only parametric system that had a \( y \) function with horizontal asymptotes.