Instructions

Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted. Turn this question sheet in with your blue book.

Time allowed: 50 minutes

Some useful trigonometric identities:

\[
\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]

1. Determine whether the integral is convergent or divergent. Evaluate those that are convergent.
   (i) (3 points) \( \int_{2\pi}^{\infty} \sin x \, dx \)  
   (ii) (3 points) \( \int_2^5 \frac{1}{\sqrt{x-2}} \, dx \)

2. (5 points) Use the Comparison Test to determine whether the integral \( \int_1^{\infty} \frac{2 + e^{-x}}{x} \, dx \) is convergent or divergent.

3. (8 points) Find the volume of the solid obtained by rotating the region enclosed by the curves \( y = x \) and \( y = x^2 \) about the line \( x = -1 \).

4. (8 points) Find the volume of the solid whose base is the triangular region with vertices \( (0, 0), (1, 0), \) and \( (0, 1) \) and whose cross-sections perpendicular to the \( y \)-axis are squares.

5. (i) (4 points) Sketch the polar curve \( r = 1 + 2 \cos \theta \).
   (ii) (8 points) Find the area of the inner loop of the limaçon in part (i).

6. A metal plate, with constant density 3 g/cm\(^2\), has a shape bounded by the curve \( y = 1 - x^2 \) and the \( x \)-axis with \( x, y \) in cm.
   (i) (3 points) Find the total mass of the plate.
   (ii) (8 points) Find the center of mass \( (\bar{x}, \bar{y}) \) of the plate.