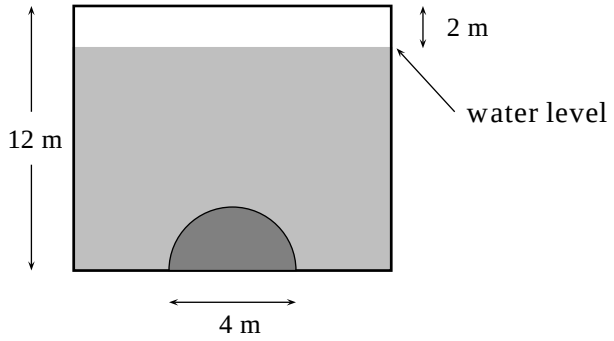
Instructions

Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted. Turn this question sheet in with your blue book.

Time allowed: 50 minutes

1. (12 points) A bucket that weighs 4 lb and a rope of neglible weight are used to draw water from a well that is 80 ft deep. The bucket is filled with 40 lb of water and is pulled up at a rate of 2 ft/s, but water leaks out of a hole in the bucket at a rate of 0.2 lb/s. Find the work done in pulling the bucket to the top of the well.

2. (15 points) A vertical dam has a semicircular gate as shown in the figure. Find the hydrostatic force against the gate.



*N*ote. The following result maybe of use:

$$\int \sqrt{a^2 \pm x^2}\,\mathrm{d}x = \frac{1}{2}x\sqrt{a^2 \pm x^2} + \frac{1}{2}a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}}\,\mathrm{d}x.$$

3. (7 points) Use the Integral Test to determine whether the series

$$\sum_{n=1}^{\infty} ne^{-n}$$

is convergent or divergent.

4. (5 points) Use the Ratio Test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

is convergent or divergent.

5. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

(i) (3 points) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n^4}$ (ii) (8 points) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$