Midterm Exam
Math 22 Spring 2008

Please be sure to include your name, student identification number, and discussion section number on the cover page of your blue book.

In the problems that follow, please show all work. Be concise but also clear. Note that problems 1-6 are worth 15 points, while problem 7 is worth 10 points. Good luck!

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1. (15 points) Write the correct form of the partial fraction decomposition of the rational function below. Find the numerator of the term whose denominator is $(x+1)^2$.

$$\frac{x^2+x+3}{(x+1)^2(x^2+5)}$$

2. (15 points) Determine which of the following expressions is correct if one uses Simpson's Rule to approximate the integral $\int_0^3 f(x)dx$ with $n=4$.

   (a) $\frac{1}{4}\left[f(\frac{3}{8})+4f(\frac{9}{8})+2f(\frac{15}{8})+4f(\frac{21}{8})\right]$
   (b) $\frac{1}{4}\left[f(0)+4f(\frac{3}{4})+2f(\frac{3}{2})+4f(\frac{9}{4})+f(3)\right]$
   (c) $\frac{3}{8}\left[f(0)+2f(\frac{3}{4})+2f(\frac{3}{2})+2f(\frac{9}{4})+f(3)\right]$
   (d) $\frac{3}{4}\left[f(0)+f(\frac{3}{4})+f(\frac{3}{2})+f(\frac{9}{4})\right]$

3. (15 points) Determine whether the following improper integral converges. If it does converge, compute the value of the integral.

$$\int_0^\infty x^2e^{-x}dx$$

4. (15 points) Sketch the finite region bounded by the two curves below. Find the area of this region.

$$x=y^2-1,\ x=1-y^4$$

5. (15 points) The integral $\int_0^\pi \pi \sin^2 x dx$ can be interpreted as the volume of a solid of revolution. Using a geometric interpretation, find the integral in the list below which yields the same value. Note that it is possible to do this problem without computing any integral.

(a) $2\int_0^1 2\pi y(\frac{\pi}{2} - \sin^{-1} y)dy$

(b) $\int_0^\pi 2\pi x \sin x dx$

(c) $\int_0^\pi \sin x dx$

(d) $\int_0^1 \pi \left[\left(\pi - \sin^{-1} y\right)^2 - \left(\sin^{-1} y\right)^2 \right] dy$

6. (15 points) Suppose the curve $\mathcal{C}$ can be represented in two ways: $y = f(x)$, $a \leq x \leq b$; or $x = g(y)$, $c \leq y \leq d$. Let $\mathcal{S}$ be the surface of revolution obtained by rotating $\mathcal{C}$ around the x-axis. Which *two* of the integrals below gives the surface area of $\mathcal{S}$?

(a) $\int_c^d 2\pi g(y)\sqrt{1 + g'(y)^2}\, dy$

(b) $\int_a^b 2\pi x\sqrt{1 + f'(x)^2}\, dx$

(c) $\int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2}\, dx$

(d) $\int_c^d 2\pi y\sqrt{1 + g'(y)^2}\, dy$

7. (10 points) Write out, but do not evaluate, the integral whose value is the length of the curve $x = e^{-y^2}$, $0 \leq y \leq 1$.