Instructions. Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted.

1. (12 points) Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = 4x^2 - xy + 4y^2$ subject to the constraint $x^2 + y^2 \leq 2$.

2. (5 points) Evaluate the integral $\int_0^3 \int_{y^2}^9 y \sin(x^2)\, dx\, dy$ by reversing the order of integration.

3. (7 points) Find the volume of the region bounded by the planes $z = 3y,\ z = y,\ y = 1,\ x = 1,$ and $x = 2$.

4. (6 points) Convert the integral $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} xy\, dx\, dy$ to polar coordinates and evaluate.

5. (7 points) Use spherical coordinates to find the volume of the region above the cone $z = \sqrt{3x^2 + 3y^2}$ and below the sphere of radius 2 centered at the origin.

6. (i) (3 points) Show that the vector field $\vec{F}(x,y) = y \cos x \vec{i} + \sin x \vec{j}$ is a gradient field.

   (ii) (3 points) Use the Fundamental Theorem of Line Integrals to calculate $\int_C \vec{F} \cdot d\vec{r}$ where C is the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$.

7. (7 points) Find $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = -y\vec{i} + x\vec{j}$ and C is the part of the ellipse $x^2 + 4y^2 = 4$ joining the point $(0,1)$ to the point $(2,0)$ in the clockwise direction.