Instructions. Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted.

1. Let $f(x, y, z) = xe^y + \ln(xz)$.

   (i) (5 points) Find the directional derivative of f at $(1, 0, 1)$ in the direction of $2\vec{i} + 2\vec{j} + \vec{k}$.

   (ii) (3 points) What is the direction of maximum rate of change of f at $(1, 0, 1)$?

2. (5 points) Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$$

3. (6 points) Use the chain rule to find $\partial z/\partial u$ and $\partial z/\partial v$ where

$$z = \tan(x + y) \quad \text{with } x = u\cos v \text{ and } y = u \sin v$$

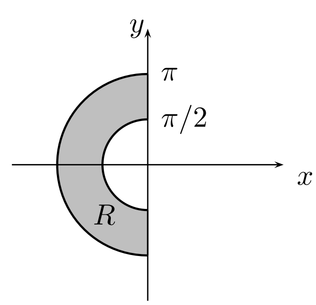
4. (7 points) Find an equation for the tangent plane to the ellipsoid $x^2 + 2y^2 + 4z^2 = 4$ at the point $(1, 1, -1/2)$.

5. (10 points) Use Lagrange multipliers to find the maximum value of $f(x, y) = x^2 - xy + y^2$ subject to the constraint $x^2 + y^2 = 1$.

6. (10 points) Evaluate the integral $\int_0^8 \int_{\sqrt[3]{y}}^2 \sqrt{x^4 + 1}\, dx\, dy$ by reversing the order of integration.

7. (10 points) Find the volume of the region bounded by the plane $x + y + z = 1$ and the three coordinate planes $x = 0,\ y = 0$, and $z = 0$.

8. (7 points) Evaluate the double integral $\iint_R \sin(x^2 + y^2)\, dA$ where R is the region below.



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9. (12 points) Use spherical coordinates to evaluate the triple integral $\iiint_W (1 + x + y)\, dV$ where W is the region bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

10. (5 points) Decide if the vector field $\vec{F}(x, y, z) = \frac{1}{x}\vec{i} + \frac{1}{y}\vec{j} + \frac{1}{xy}\vec{k}$ is a gradient field. If so, find the potential function. If not, explain why not.

11. (10 points) Find $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = \ln y\vec{i} + \ln x\vec{j}$ and C is the curve $y = x^3/8$ from $(4, 8)$ to $(8, 64)$.

12. (10 points) Use Green's theorem to find the line integral of $\vec{F} = 3y\vec{i} + xy\vec{j}$ around the unit circle oriented counterclockwise.