Instructions. Attempt all questions. Answers must be justified in order to gain full credit. Calculators are not permitted.

1. Let \( f(x, y, z) = xe^y + \ln(xz) \).
   (i) (5 points) Find the directional derivative of \( f \) at \((1, 0, 1)\) in the direction of \(2\hat{i} + 2\hat{j} + \hat{k}\).
   (ii) (3 points) What is the direction of maximum rate of change of \( f \) at \((1, 0, 1)\)?

2. (5 points) Show that the following limit does not exist:
   \[
   \lim_{(x,y) \to (0,0)} \frac{x^2y}{x^4 + y^2}
   \]

3. (6 points) Use the chain rule to find \( \frac{\partial z}{\partial u} \) and \( \frac{\partial z}{\partial v} \) where
   \[ z = \tan(x + y) \quad \text{with} \; x = u \cos v \; \text{and} \; y = u \sin v \]

4. (7 points) Find an equation for the tangent plane to the ellipsoid \( x^2 + 2y^2 + 4z^2 = 4 \) at the point \((1, 1, -1/2)\).

5. (10 points) Use Lagrange multipliers to find the maximum value of \( f(x, y) = x^2 - xy + y^2 \) subject to the constraint \( x^2 + y^2 = 1 \).

6. (10 points) Evaluate the integral \( \int_0^8 \int_{\sqrt[3]{y}}^2 \sqrt{x^4 + 1} \, dx \, dy \) by reversing the order of integration.

7. (10 points) Find the volume of the region bounded by the plane \( x + y + z = 1 \) and the three coordinate planes \( x = 0 \), \( y = 0 \), and \( z = 0 \).

8. (7 points) Evaluate the double integral \( \int \int_R \sin(x^2 + y^2) \, dA \) where \( R \) is the region below.
9. (12 points) Use spherical coordinates to evaluate the triple integral \( \iiint_{W} (1 + x + y) \, dV \) where \( W \) is the region bounded by the paraboloid \( z = 4 - x^2 - y^2 \) and the \( xy \)-plane.

10. (5 points) Decide if the vector field \( \vec{F}(x, y, z) = \frac{1}{x} \vec{i} + \frac{1}{y} \vec{j} + \frac{1}{xy} \vec{k} \) is a gradient field. If so, find the potential function. If not, explain why not.

11. (10 points) Find \( \int_{C} \vec{F} \cdot d\vec{r} \) where \( \vec{F}(x, y) = \ln y \vec{i} + \ln x \vec{j} \) and \( C \) is the curve \( y = x^3/8 \) from \((4, 8)\) to \((8, 64)\).

12. (10 points) Use Green’s theorem to find the line integral of \( \vec{F} = 3y \vec{i} + xy \vec{j} \) around the unit circle oriented counterclockwise.