MATH 23 – Midterm 1 *Fall Semester 2007*

Duration: 50 minutes

Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 70.

1. (13 pts: 7, 6) $f(x, y) = \sqrt{16 - 4x^2 - y^2}$.
   (a) Draw a contour map of f showing at least three level curves. Remember to label your axes and level curves.
   (b) Find a vector function (or parametric equations) that represents the intersection curve of the graph $z = f(x, y)$ and the plane $x = 1$.
2. (15 pts: 2, 2, 5, 3, 3) Consider the plane Π: $2x - y + 3z = 0$ and the vector $\vec{v} =< 2, -2, 4 >$.
   (a) Find a normal vector $\vec{n}$ to the plane Π.
   (b) Does the plane π pass through the origin? Why?
   (c) Find $\text{proj}_{\vec{n}}\vec{v}$, the vector projection of $\vec{v}$ onto $\vec{n}$ from part (a). (If you cannot solve part (a), use $\vec{n} =< 1, 0, 3 >$.)
   (d) Find the distance between the point $(2, -2, 4)$ and the plane Π.
   (e) What can you say about the direction of $\vec{v} - \text{proj}_{\vec{n}}\vec{v}$?
3. (15 pts: 5 each) In a contour map of the function $f(x, y)$, the point $(0, 2)$ lies on the level curve $f(x, y) = 5$. We also know that $f_x(0, 2) = -3$ and $f_y(0, 2) = 4$.
   (a) Find the direction in which $f(x, y)$ increases fastest at $(0, 2)$, and find the maximum rate of increase.
   (b) Find one tangent vector to the level curve $f(x, y) = 5$ at $(0, 2)$.
   (c) Find an equation of the tangent plane to the graph $z = f(x, y)$ at the point above $(0, 2)$.
4. (15 pts: 10, 5) Consider the function $f(x, y) = x - x^2 - y^2$.
   (a) Find and classify all critical points of $f(x, y)$.
   (b) Find the absolute maximum and absolute minimum values of $f(x, y)$ over $D = \{ (x, y) \,|\, x^2 + y^2 \leq 1 \}$.
5. (12 pts: 3 each) Answer the following questions in no more than two lines of text.
   (a) Is it possible for a function f to have $f_x(x, y) = 3x^2 - y$ and $f_y(x, y) = x^3 - 1$ as partial derivatives? Explain why.
   (b) Write down a vector function $\vec{r}(t)$ (or parametric equations) for a **space** curve whose curvature is zero everywhere.
   (c) If $B(s, r)$ is the price of burritos, s the price of beans and r the price of rice, what is the meaning of $\partial B/\partial s$?
   (d) If you know that $\lim_{x\to 0} f(x, mx) = \lim_{x\to 0} f(x, kx^2) = 2$, what can you conclude about $\lim_{(x,y)\to(0,0)} f(x, y)$?