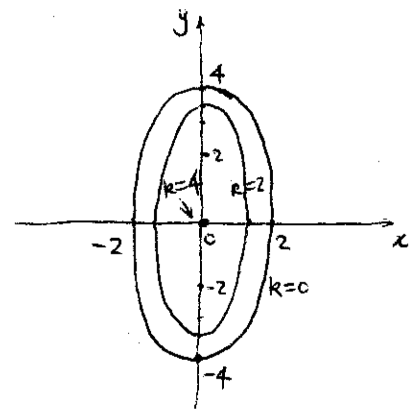
Math 23 Midterm 1 Fall 2007

1. (a) $k = \sqrt{16-4x^2-y^2} \Rightarrow k^2 = 16-4x^2-y^2 \Rightarrow 16-k^2 = 4x^2+y^2 \quad (0 \le k \le 4)$

$k=0 \; : \quad 16 = 4x^2+y^2 \Rightarrow 1 = \frac{x^2}{4} + \frac{y^2}{16}$

$k=2 \; : \quad 12 = 4x^2+y^2 \Rightarrow 1 = \frac{x^2}{3} + \frac{y^2}{12}$

$k=4 \; : \quad 0 = 4x^2+y^2 \Rightarrow (x,y)=(0,0)$



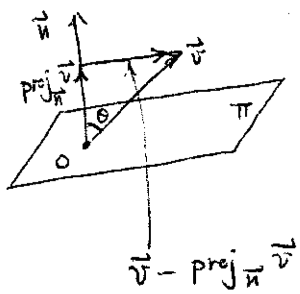
(b) $x=1 \Rightarrow z = f(1,y) = \sqrt{16-4\cdot 1^2-y^2} = \sqrt{12-y^2}$

$\Rightarrow \boxed{\vec{r}(t) = \langle 1, \; t, \; \sqrt{12-t^2} \rangle}$ or $\boxed{x=1, \; y=t, \; z=\sqrt{12-t^2}}$

2. (a) $\boxed{\vec{n} = \langle 2, -1, 3 \rangle}$

(b) Yes, because $2(0)-(0)+3(0)=0$.

(c)



$\text{proj}_{\vec{n}}\vec{v} = |\vec{v}|\cos\theta \, \frac{\vec{n}}{|\vec{n}|} = \cancel{|\vec{v}|} \, \frac{\vec{v}\cdot\vec{n}}{\cancel{|\vec{v}|}|\vec{n}|} \, \frac{\vec{n}}{|\vec{n}|}$

$= \frac{\vec{v}\cdot\vec{n}}{|\vec{n}|^2} \, \vec{n}$

$= \frac{4+2+12}{4+1+9} \langle 2,-1,3 \rangle$

$= \frac{18}{14} \langle 2,-1,3 \rangle$

$= \boxed{\frac{9}{7} \langle 2,-1,3 \rangle}$

(d) distance $= |\text{proj}_{\vec{n}}\vec{v}| = \frac{9}{7}\sqrt{4+1+9} = \boxed{\frac{9}{7}\sqrt{14}}$

(e) $\vec{v} - \text{proj}_{\vec{n}}\vec{v}$ is parallel to π.

3 (a) f increases fastest at $(0,2)$ in the direction of

$$\nabla f(0,2) = \langle f_x(0,2), f_y(0,2)\rangle = \boxed{\langle -3, 4\rangle}$$

~~(b)~~ Maximum rate of increase $= |\nabla f(0,2)| = \sqrt{9+16} = \boxed{5}$

(b) $\nabla f \perp$ level curves, so one tangent vector to level curve could be

$\boxed{\langle 4, 3\rangle}$

(c) $z-5 = -3(x-0)+4(y-2) \Rightarrow z = 5-3x+4y-8 \Rightarrow \boxed{z=-3x+4y-3}$

4 (a) $0 = f_x = 1-2x \Rightarrow x = \frac{1}{2}$

$0 = f_y = -2y \Rightarrow y = 0$ $\Rightarrow$ critical pt $\boxed{(\frac{1}{2}, 0)}$

$f_{xx} = -2 < 0, \quad f_{yy} = -2, \quad f_{xy} = 0 = f_{yx}$

$D = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 > 0 \Rightarrow (\frac{1}{2}, 0)$ is a $\boxed{\text{local max}}$

(b) inside D : $f(\frac{1}{2}, 0) = \frac{1}{2} - \frac{1}{4} - 0 = \frac{1}{4}$

boundary $x^2+y^2=1$: $f(x,y) = x-1$ linear, so absolute

max/min happen at boundary $x = \pm 1$. $x=1 \Rightarrow f(1,0)=0$,

$x=-1 \Rightarrow f(-1,0) = -2$

$\Rightarrow$ absolute max. value $= \boxed{\frac{1}{4}}$

absolute min. value $= \boxed{-2}$

5 (a) No. Because $f_{xy} = -1$, $f_{yx} = 3x^2$, both are continuous,

but not equal.

(b) $\vec{r}(t) = \langle 1, t, 2t\rangle$ (Any straight line would do.)

(c) $\frac{\partial B}{\partial S}$ is the rate of change of Burrito price with respect to the price of beans.

(d) We don't know whether $\lim_{(x,y)\to(0,0)} f(x,y)$ exists or not. If it does, it is equal to 2.