1. (10 pts: 5 each) The mass $M$ of a thin plate $D$ in the $xy$–plane can be calculated as

$$M = \int_{0}^{1} \int_{0}^{2\sqrt{x}} \rho(x, y) \, dy \, dx,$$

where $\rho(x, y)$ is the density function.

(a) Sketch the shape of this thin plate $D$.

(b) Change the order of integration.

2. (15 pts: 5 each) A solid $V$ lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 2$. Set up, but do not evaluate, iterated integrals to find the volume of $V$ in the following coordinate systems.

(a) The Cartesian (rectangular) coordinates.

(b) The cylindrical coordinates.

(c) The spherical coordinates.

3. (10 pts: 5 each) $\vec{F}(x, y) = y \cos x \vec{i} + \sin x \vec{j}$ is a conservative vector field.

(a) Find a potential function $f$ for $\vec{F}$. (That is, find a scalar function $f(x, y)$ such that $\vec{F} = \nabla f$.

(b) Use the Fundamental Theorem for Line Integrals to calculate $\int_{C} \vec{F} \cdot d\vec{r}$, where $C$ the the part of the curve $y = \sin x$ going from $(0, 0)$ to $(3\pi/2, -1)$.
4. (25 pts total) Consider the following vector field $\vec{F}$, oriented surfaces $S_1$ and $S_2$, and curve $C$.

$$\vec{F}(x, y, z) = z\vec{i} - 2y\vec{j} - (x - 2z)\vec{k},$$

$S_1$ = the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the $xy$–plane,

$S_2$ = the part of the $xy$–plane that lies within the paraboloid $z = 1 - x^2 - y^2$,

$C$ = the common boundary of $S_1$ and $S_2$.

Both $S_1$ and $S_2$ are oriented upward and $C$ is oriented positively. (You do not need to answer the following questions in order.)

(a) (5 pts) Find the curl and divergence of $\vec{F}$.
(b) (7 pts) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ by parametrizing the curve.
(c) (8 pts) Evaluate the surface integral $\iint_{S_1} \text{curl} \vec{F} \cdot d\vec{S}$ by parametrizing the surface.
(d) (2 pts) Should you expect to have the same result from parts (b) and (c)? Why?
(e) (3 pts) Using the divergence theorem, explain why $\iint_{S_1} \vec{F} \cdot d\vec{S} = \iint_{S_2} \vec{F} \cdot d\vec{S}$. 

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\begin{align*}
S_1 &: \ z = 1 - x^2 - y^2 \\
S_2 &: \\
C &\
\end{align*}
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