Duration: 50 minutes

Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 60.

1. (10 pts: 5 each) The mass M of a thin plate D in the xy–plane can be calculated as

$$M = \int_0^1 \int_0^{2\sqrt{x}} \rho(x, y)\, dy\, dx,$$

where $\rho(x, y)$ is the density function.

   (a) Sketch the shape of this thin plate D.

   (b) Change the order of integration.

2. (15 pts: 5 each) A solid V lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 2$. Set up, but **do not evaluate**, iterated integrals to find the volume of V in the following coordinate systems.

   (a) The Cartesian (rectangular) coordinates.

   (b) The cylindrical coordinates.

   (c) The spherical coordinates.

3. (10 pts: 5 each) $\vec{F}(x, y) = y \cos x \vec{i} + \sin x \vec{j}$ is a conservative vector field.

   (a) Find a potential function f for $\vec{F}$. (That is, find a scalar function $f(x, y)$ such that $\vec{F} = \nabla f$.

   (b) Use the Fundamental Theorem for Line Integrals to calculate $\int_C \vec{F} \cdot d\vec{r}$, where C the the part of the curve $y = \sin x$ going from $(0, 0)$ to $(3\pi/2, -1)$.

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4. (25 pts total) Consider the following vector field $\vec{F}$, oriented surfaces S_1 and S_2, and curve C.

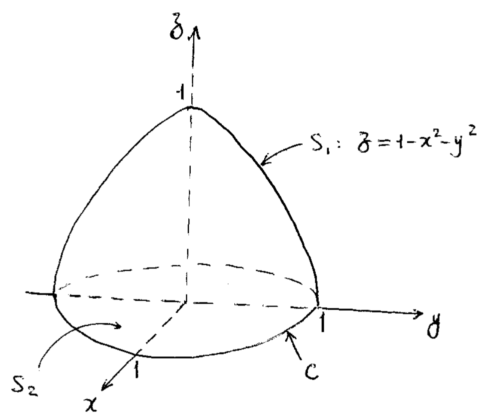
$$\vec{F}(x,y,z) = z\vec{i} - 2y\vec{j} - (x-2z)\vec{k},$$

$S_1 =$ the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy–plane,

$S_2 =$ the part of the xy–plane that lies within the paraboloid $z = 1 - x^2 - y^2$,

$C =$ the common boundary of S_1 and S_2.

Both S_1 and S_2 are oriented upward and C is oriented positively. (You do not need to answer the following questions in order.)



(a) (5 pts) Find the curl and divergence of $\vec{F}$.

(b) (7 pts) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ by parametrizing the curve.

(c) (8 pts) Evaluate the surface integral $\iint_{S_1} \operatorname{curl} \vec{F} \cdot d\vec{S}$ by parametrizing the surface.

(d) (2 pts) Should you expect to have the same result from parts (b) and (c)? Why?

(e) (3 pts) Using the divergence theorem, explain why $\iint_{S_1} \vec{F} \cdot d\vec{S} = \iint_{S_2} \vec{F} \cdot d\vec{S}$.