Duration: 3 hours
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 100.

1. (20 pts: 2 each)
   (a) Write down two properties of the gradient ∇f of a function $f(x, y, z)$.
   (b) If you know that $\lim_{x\to 0} f(x, mx) = \lim_{x\to 0} f(x, kx^2)$, what can you conclude about the continuity of $f(x, y)$ at $(0, 0)$?
   (c) Give an example of a two–variable function $f(x, y)$ (a formula, a sketch, or a word description) that is continuous but not differentiable at the origin.
   (d) Let f be a scalar field and $\vec{F}$ a vector field. Which of the following expressions are meaningful?

(i) grad $\vec{F}$	(ii) curl $\vec{F}$	(iii) div f
(iv) grad(div $\vec{F}$)	(v) curl(grad f)	(vi) curl(curl $\vec{F}$)

   (e) Write down the formula for the average value of a scalar function $f(x, y, z)$ over a solid region E in space.
   (f) Given $\vec{F} = \dfrac{-y\vec{i} + x\vec{j}}{x^2 + y^2}$, what condition do we have to impose on a smooth simple closed curve C so that we can use Green's Theorem to calculate $\int_C \vec{F} \cdot d\vec{r}$?
   (g) If a transformation T is given by $x = u - v$, $y = uv$, what is the Jacobian of T?
   (h) Does there exist a vector field $\vec{F}$ such that curl $\vec{F} = x\vec{i} + \vec{j} + z\vec{k}$? Why or why not?
   (i) Given a vector function $\vec{r}(t)$, how do you check whether or not it is parametrized by arc length?
   (j) If all component functions of $\vec{F}$ have continuous partial derivatives on $\mathbb{R}^3$, and if $\oint_C \vec{F} \cdot d\vec{r} = 0$ for any closed space curve C, what can you say about $\vec{F}$?

2. (8 pts) Consider three points in space $P(3, -1, 1)$, $Q(4, 0, 2)$, and $R(5, -1, -1)$
   (a) Find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.
   (b) Find an equation of the plane going through these three points.

3. (9 pts) The tangent plane to the graph $z = f(x, y)$ at the point above $(0, 1)$ is given by $z = 5 + x - 3y$.
   (a) What is value of $f(0, 1)$?
   (b) What is the gradient $f(0, 1)$ of $f(x, y)$ at $(0, 1)$?
   (c) What is the directional derivative of $f(x, y)$ at $(0, 1)$ in the direction $\vec{v} = -\vec{i} - \vec{j}$?
   (d) If $x(t) = \sin t$ and $y(t) = e^{2t}$, find $\frac{df}{dt}$ at $t = 0$.

continuted on the back $\longrightarrow$

4. (9 pts) Consider the function $f(x, y) = x^2 - 2y + y^2$.
   (a) Find and classify all critical points of $f(x, y)$.
   (b) Find the absolute maximum and absolute minimum values of $f(x, y)$ over $D = \{\, (x, y) \,|\, x^2 + y^2 \leq 4 \,\}$.

5. (9 pts) Find the volume of the solid bounded by the cylinder $x^2+y^2 = 4$ and the planes $z = 0$ and $y + z = 3$.

6. (10 pts) Consider the parametric curve

$$C : \quad x = \cos t, \qquad y = \sin t, \qquad z = t.$$

   (a) Sketch the curve C and indicate with an arrow the direction of increasing t.
   (b) Find parametric equations of the tangent line to the curve C at the point $(-1, 0, \pi)$.
   (c) Evaluate the line integral of $\vec{F}(x, y, z) = x\vec{i}+y\vec{j}+2z\vec{k}$ along C from $(1, 0, 0)$ to $(-1, 0, \pi)$.

7. (10 pts) D is a triangular region in the xy–plane with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$.
   (a) Set up **two** iterated integrals to evaluate $\iint_D f(x, y)\, dA$, one integrating x first and the other integrating y first.
   (b) Use Green's Theorem to evaluate $\oint_C (y^3 + xy)\, dx + (3xy^2)\, dy$ where C is the boundary of D oriented **clockwise**.

8. (9 pts) Let S be the part of the surface $z = xy^2$ that lies above the region $-1 \leq x \leq 0$ and $0 \leq y \leq 2$ and oriented upward.
   (a) Parametrize the surface described above.
   (b) Compute the flux of $\vec{F} = x^2/z\ \vec{i} + z/2\ \vec{j} + z/y^2\ \vec{k}$ through S.

9. (8 pts) Let C be an ellipse that lies in the plane $x + y + z = 1$. Use Stokes' theorem to show that the line integral $\oint_C z\, dx - 2x\, dy + 3y\, dz$ depends only on the area of the region enclosed by C and not on the shape of C or its location in the plane.

10. (8 pts) Let S be the top half of the sphere $x^2 + y^2 + z^2 = 1$ oriented upward, and $\vec{F} = (z^2x)\vec{i} + (\frac{1}{3}y^3 + \tan z)\vec{j} + (x^2z + 1)\vec{k}$
   (a) Explain how you could use the divergence theorem to calculate the flux of $\vec{F}$ across S $\iint_S \vec{F} \cdot d\vec{S}$.
   (b) Use the divergence theorem to compute the flux through S as you explained.

HAVE A NICE HOLIDAY!!