MATH 23 – Midterm 2 *Spring Semester 2007*

Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 100.

1. (20 pts) Consider the domain R in the xy-plane such that $0 \le y \le 1$ and $0 \le x \le 2$ and $x^2 + y^2 \ge 4$.
   (a) Draw this domain.
   (b) Setup 2 integrals to evaluate the volume between a function $f(x, y) > 0$ and the plane $z = 0$ over R, one integrating x first and the other integrating y first.
   (c) Evaluate the volume above R and below the surface $z = xy$.
2. (18 pts) A drill bit is shaped like the inside of the cone $z^2 = x^2 + y^2$ for 0cm$\le z \le$ 2cm. Write down an integral describing the mass of this drill bit if its density is $d(x, y, z) = \frac{300}{x^2+y^2+2z^2+100}$ g/cm^3 in **TWO of the following THREE coordinate systems**.
   (a) Cartesian coordinates.
   (b) Cylindrical coordinates.
   (c) Spherical coordinates.
3. (20 pts) Consider the force field $\vec{F}(x, y) = 1/x\,\vec{i} + 2/y\,\vec{j}$ and the curve C, a parabola going from $(1, 1)$ to $(3, 9)$ via the point $(2, 4)$
   (a) Compute the work of $\vec{F}(x, y)$ done on a particle traveling along C by parametrizing the curve.
   (b) Find the potential $\phi(x, y)$ such that $\nabla\phi = \vec{F}$ and use it to verify your answer to part a).
   (c) Could you use Green's theorem to find the work done by $\vec{F}$ on a particle going counterclockwise around the circle of radius 1 centered at the origin? Justify your answer.
4. (18 pts) Consider the lower half of the sphere of radius 2m centered at the origin
   (a) Parametrize the surface described above.
   (b) Compute the flux of $\vec{F} = (1/x\,\vec{i} + 3/y\,\vec{j} + x^2\vec{k})$m/min through the half sphere oriented outward.
   (c) Assume the flux of $\vec{F}$ through a surface S is 2m^3/s. If the surface describes a gold-digger's pan and $\vec{F}$ is the velocity of water containing 0.001 ounces of gold per meter cubed, how long would it take to gather 4 ounces of gold?
5. (24 pts) Answer the following questions in no more than two lines of text or formulas (much less is usually needed if you are right on point).
   (a) What is a formula for the average height of a surface $z = f(x, y)$ over a domain R in the xy-plane?
   (b) If $d(x, y)$ is the density of fairy shrimp per unit length of a waterway, what does $\int_C d(x, y)dl$ represent if C is the path of the waterway?
   (c) For a given velocity field $\vec{F}(x, y, z)$, how would you orient a surface S to maximize the flux through that same surface?
   (d) Sketch or describe a 2-dimensional vector field, $\vec{F}$, for which curl $\vec{F} = 0$ inside the circle of radius 1 centered at the origin and curl $\vec{F} = -1$ everywhere else.
   (e) Sketch or describe the curve parametrized by
   $$x = t, \qquad y = e^{-t}\cos t, \qquad z = e^{-t}\sin t$$
   for $0 \le t \le \infty$.
   (f) If $\vec{r}(s, t) = x(s, t)\,\vec{i} + y(s, t)\,\vec{j} + z(s, t)\,\vec{k}$, is the parametrization of a certain surface, what can you say about the length and direction of the vector $(\vec{r}_t \times \vec{r}_s)\Delta s\Delta t$?