MATH 23 – Practice Midterm 2 *Spring Semester 2007*

Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 100.

1. (20 pts) Consider the domain R in the xy-plane such that $1 \leq x \leq 2 + \cos y$ and $0 \leq y \leq \pi$.
   (a) Draw this domain.
   (b) Set up 2 integrals to evaluate the volume between a function $f(x, y) > 0$ and the plane $z = 0$ over R: one integrating x first and the second integrating y first.
   (c) Evaluate the volume above R and below the surface $z = e^x \sin y$.

2. (18 pts) A spherical orange of radius 4cm and with center at the origin, is sliced in eight equal parts by cutting it vertically along the x and y axis and along the planes $y = x$ and $y = -x$. Write the integral you would use to compute the mass of the slice of orange between the $y = x$ plane and the y-axis if the density of the orange is $d(x, y, z) = 0.5 + 0.2(x^2 + y^2)/(z^2 + 1)$.
   (a) In cylindrical coordinates.
   (b) In spherical coordinates.

3. (20 pts) Consider the vector field $\vec{F} = x^2 y^3 \vec{i} + 0\vec{j}$ and the curve C describing the boundary of a square of side 2 centered at the origin with sides parallel to the axes.
   (a) Compute the line integral of $\vec{F}$ over C by parametrizing the curve.
   (b) Can you use Green's theorem to compute this line integral? Why or why not?
   (c) If you can use Green's theorem, do so, if not suggest a simple modification to the problem that would allow you to use it.

4. (18 pts) Consider the cylinder of radius 3 centered on the x-axis for $0 \leq x \leq 2$.
   (a) Draw the cylinder and parametrize its surface.
   (b) Compute the flux of $\vec{F} = yz\vec{i} + xy\vec{j} + xz\vec{k}$ into that cylinder.
   (c) If $\vec{F}$ describe the velocity field of flowing water in m/s and the cylinder has radius 3m, what does the previous calculation describe, in non-mathematical terms?

5. (24 pts) Answer the following questions in no more than two lines of text (much less is usually needed if you are right on point).
   (a) If a vector field $\vec{F}$ is such that its circulation around any closed loop is 0, what can you say about the line integral between two points $P = (x_0, y_0)$ and $Q = (x_1, y_1)$ along a straight line compared to that between the same points along a path twice a long?
   (b) When do you use the Jacobian $\frac{\partial(x,y)}{\partial(s,t)}$ of a transformation $x(s, t)$ and $y(s, t)$ from the (x, y) coordinates to the (s, t) coordinates?
   (c) Describe or sketch the surface parametrized by $x = s$, $y = s \sin t$, $z = s \cos t$.
   (d) If $\vec{F}$ is the velocity field of the wind and the air contains 0.1 grams of pollen per meter cubed, how much pollen would you find in a surface S after 30 minutes if $\int\int_S \vec{F} \cdot \vec{n} dA = 6\text{m}^3/\text{min}$?
   (e) Find a 3-dimensional vector field $\vec{F}(x, y, z)$ where each vector has length 2 and points towards the point $(2, 3, 4)$ (except at $(2, 3, 4)$ where $\vec{F}(2, 3, 4) = 0\vec{i} + 0\vec{j} + 0\vec{k}$).
   (f) Write a formula to compute the average value of $g(x, y, z)$ over a three dimensional domain V.