Duration: 180 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 100.

1. (20 pts) Answer the following questions in no more than two lines of text or formulas (much less is usually needed if you are right on point).
   (a) How do you verify that two vectors $\vec{v}$ and $\vec{w}$ are perpendicular?
   (b) If $\vec{c} = \vec{a} \times \vec{b}$, what can you say about the length and direction of $\vec{c}$?
   (c) Give two properties of the gradient ∇f of a function $f(x, y, z)$.
   (d) Sketch, name or describe a function that is discontinuous at the origin and a function that is continuous but not differentiable at the origin.
   (e) If a contour of $f(x, y)$ is given by $y^2 + x^3 = 1$, find a point where $f(x, y) = f(0, 1)$ (other than $(0, 1)$, of course).
   (f) How would you verify whether a vector field $\vec{F}$ is a gradient field (conservative) or not?
   (g) If S is a surface and $\vec{F}$ a vector field, when can you use the divergence theorem directly to calculate the flux of $\vec{F}$ through S?
   (h) Sketch or describe in words a vector field with a positive curl everywhere (a formula is not sufficient)4
   (i) Give a parametrization of the line going from $(-1, -1, 3)$ to $(0, 1, 4)$.
   (j) Which of the following are vectors?
      (i) A velocity field ($\vec{u}$) (ii) $\vec{a} \cdot \vec{b}$
      (iii) The divergence of a velocity field (div$\vec{F}$) (iv) $\vec{a} \times \vec{b}$
      (v) The curl of a three dimensional velocity field (curl$\vec{F}$) (vi) The gradient of a function (∇f)
2. (9 pts) Given the implicit function $z^2 + 9x^2 + y^2/4 = 1$
   (a) Draw at least 2 cross-sections of this surface by keeping x fixed (specify the value of x).
   (b) Draw at least 2 contours.
   (c) **SKETCH** the surface in a manner consistent with what you found above.
3. (8 pts) Consider the following three points in space $m_1 = (-1, 0, 2)$, $m_2 = (1, 4, 2)$ and $m_3 = (0, 2, 1)$
   (a) Find the vectors $\vec{v}_1$ going from m_1 to m_2 and $\vec{v}_2$ going from m_1 to m_3.
   (b) Using $\vec{v}_1$ and $\vec{v}_2$, find the equation of the plane going through these three points.
4. (9 pts) Above the point $(-1, 2)$ in the xy-plane, the plane tangent to the function $f(x, y) = xy$ is labeled $p(x, y)$.
   (a) What is $p(-1, 2)$?
   (b) What is the equation of $p(x, y)$?
   (c) If x and y are functions of time $x(t) = -t^2$ and $y(t) = 2\cos(t - 1)$, use the chain rule to compute $\frac{df}{dt}$ at $t = 1$.

SEE BACK

5. (9 pts) Consider the function $f(x, y) = 2xy^2 - x^2 - 32y$.
   (a) Find **AND** classify all the critical points of $f(x, y)$.
   (b) How would you determine if $f(x, y)$ has a global maximum over $D = \{\text{all } x \leq -2 \text{ and all } y \geq 1\}$?

6. (10 pts) Consider the domain R in the xy-plane such that $0 \leq y \leq 4$ and $0 \leq x \leq 2$ and $y \leq x^2$.
   (a) Draw this domain.
   (b) Set up 2 integrals to evaluate the volume over R between **TWO** functions $f(x, y)$ and $g(x, y)$, with $f(x, y) > g(x, y)$, one integrating x first and the other integrating y first.
   (c) Evaluate the volume above R and between the surfaces $z = xy$ and $z = -1$.

7. (8 pts) The bottom of a silo is shaped like the cylinder $x^2 + y^2 = 9$ for $-3 \leq z \leq 0$ and the cap of the silo is a the half-sphere $x^2 + y^2 + z^2 = 9$ for $z \geq 0$. The density of the grain inside the silo is $d(x, y, z) = 1 + x^2 z/10$.
   (a) Find an integral expression (do not evaluate) for the mass of grain in the silo:

8. (9 pts) Consider the force field $\vec{F}(x, y) = (8xy)\,\vec{i} + (3y^2 + 2x))\,\vec{j}$ and the curve C, which is the **LOWER** half of the ellipse $4x^2 + y^2 = 1$ oriented in counter-clockwise direction.
   (a) Compute the work of $\vec{F}(x, y)$ done on a particle traveling along C by parametrizing the curve.
   (b) By symmetry, the work on the lower part of the ellipse is half of the work done by a particle going all the way around the ellipse. Use Green's theorem to set up (but not evaluate) an integral for the work in part a).

9. (10 pts) Consider the surface S given by $z = xy^2$ over the region $-1 \leq x \leq 0$ and $0 \leq y \leq 2$ and oriented with its normal pointing up.
   (a) Parametrize the surface described above.
   (b) Compute the flux of $\vec{F} = (x^2/z\,\vec{i} + z/2\,\vec{j} + z/y^2\,\vec{k})$m/s through S.
   (c) What would be the flux through the same surface oriented with its normal pointing down? (if you didn't solve part b), assume the answer was 1.32)
   (d) If the units of x and y are in meters, what are the units of the flux through S?

10. (8 pts) Consider the surface S of a filter given by part of a cone $z = \sqrt{3(x^2 + y^2)}$ (of apex angle $\pi/3$) restricted to $0 \leq z \leq \sqrt{3}$. The velocity field of air flowing through the filter is:
    $\vec{F} = (\sin y - 2xz)\vec{i} + (e^x - e^z + y)\vec{j} + (z^2 + 1)\vec{k}$
    (a) Explain how you could use the divergence theorem to compute the flux through S oriented with its normal pointing outward.
    (b) Use the divergence theorem to compute the flux through S as you explained in a).

HAVE A GOOD SUMMER!!