

**Duration: 180 minutes**

Instructions: Answer all questions, without the use of books or calculators. You may have a half sheet of 8.5X11 paper with both sides filled out. Partial credit will be awarded for correct work. You may use the back of the pages of the exam should it be necessary, but please indicate in writing that you have done so. The total number of points is 100. Please write at the top of the exam “drop” or “replace”, depending on how you wish your second midterm grade to be computed. The problems with a “\*” are those that will count toward your second midterm score should you choose the “replace” option.

Problem	Score
1	
2	
3	
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Total	

1. (10 points) Which of the following four planes are parallel? Are any of them identical?

(a)  $4x-2y+6z=12$

(b)  $-6x+3y-9z=-18$

(c)  $4x-2y-2z=6$

(d)  $z=2x-y-3$

2. (10 points) Find the parametric equations for the tangent line of the curve  $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$  at the point  $(1, 1, 1)$ .

3. (15 points\*)

- (a) Find the partial derivatives of  $z = \sqrt{3x + e^{8y}}$  at  $(x, y) = (1, 0)$ .
- (b) Use linearization and your answer in part (a) to approximate  $\sqrt{3.03 + e^{-0.4}}$ .

4. (10 points\*) The price of a box must be fixed at  $P_0$ . Suppose the top and bottom of the box cost 2 cents per square inch and the sides cost 1 cent per square inch. Find the dimensions of the box which maximize the volume.

5. (10 points\*) Compute the following double integral where  $D$  is the region bounded by the curves  $y = 2$ ,  $y = \sqrt[3]{x}$ , and  $x = 0$ .

$$\iint_D \sin(2 + y^4) dA$$

6. (10 points) Let  $\mathbf{F}$  be the two dimensional vector field given by  $\mathbf{F}(x, y) = \langle x^3, \sin y \rangle$ . Let the curve  $\mathcal{C}$  be the part of the ellipse  $x^2 + \frac{y^2}{4} = 1$  in the upper half plane and oriented counterclockwise. Find the line integral  $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ .

7. (15 points) Let  $W$  be the cylinder with radius 1 and height 4 placed in a coordinate space so that its center coincides with the  $z$ -axis and its base lies on the  $xy$ -plane. Suppose further that the density of the cylinder is given by  $\rho(x, y, z) = (1 + z) \sin(x^2 + y^2)$ . Find the center of mass of the cylinder.

8. (10 points) Let  $\mathbf{F}$  be the two dimensional vector field  $\langle \sqrt{x} + y^3, x^2 + \sqrt{y} \rangle$ . Compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $C$  is the square with vertices  $(1, 1)$ ,  $(1, -1)$ ,  $(-1, -1)$ , and  $(-1, 1)$  oriented in the counterclockwise direction.
9. (10 points) Let  $E$  be the region in three dimensional space bounded by the surfaces  $x^2 + y^2 = 1$ ,  $z = 0$ ,  $z = 2$ ,  $x = 0$ , and  $y = 0$ . Find the flux of the vector field  $\mathbf{F}(x, y, z) = \langle x^4, x^3z^2, 4xy^2z \rangle$  out through the boundary of  $E$ .