1 (a) F (b) T (c) F (d) F

2 (a) (1) FIRST-ORDER, NONLINEAR, NONAUTONOMOUS

(2) SECOND-ORDER, LINEAR, HOMOGENEOUS, VARIABLE COEF., NON AUTONOMOUS

(b) $$\frac{2y}{y^2+1}\frac{dy}{dt} = \frac{1}{t} \Rightarrow \int \frac{2y}{y^2+1}\,dy = \int \frac{1}{t}\,dt$$

LHS: LET $u = y^2+1$, $\frac{du}{dt} = 2y$ $\Big\}$ $\int \frac{2y}{y^2+1}\,dy = \int \frac{1}{u}\,du = \ln|u| = \ln|y^2+1|$

THUS

$$\ln|y^2+1| = \ln|t| + \tilde{C} \qquad y^2+1 = Ct$$

$$y(1) = 1 \Rightarrow C = 2 \qquad \boxed{y = \sqrt{2t-1}}$$

LET $f(y,t) = \frac{1+y^2}{2y}\frac{1}{t}$ CLEARLY $f(y,t)$ IS CONT. AT $(1,1)$

$$\frac{\partial f}{\partial y} = \frac{1}{2t}\frac{\partial}{\partial y}\left(\frac{1}{y} + y\right) = \frac{1}{2t}\left(-y^{-2}+1\right)$$ $\frac{\partial f}{\partial y}$ IS ALSO CONT AT $(1,1)$

BY PICARD'S TH'M SOL'N IS UNIQUE ABOUT $(1,1)$

(c) STANDARD LINEAR FORM

$$\frac{dy}{dt} \underbrace{- \frac{\sin(t)}{\cos(t)}}_{p(x)} y = \sin(t) \qquad \mu(t) = \text{EXP}\left[\int p(t)\,dt\right]$$

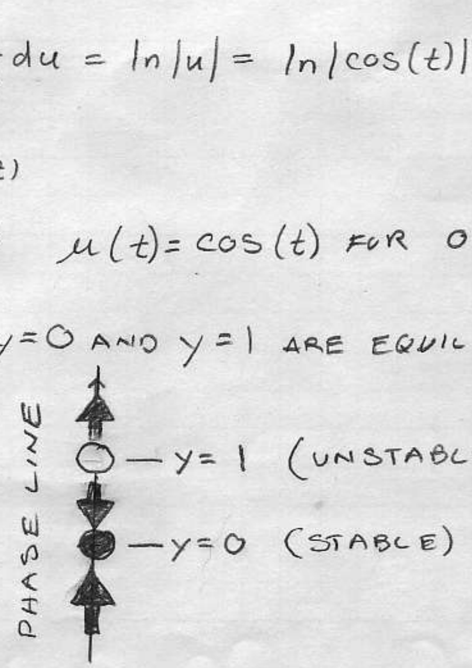
$$\int p(t)\,dt = \int \frac{-\sin(t)}{\cos(t)}\,dt = \int \frac{1}{u}\,du = \ln|u| = \ln|\cos(t)|$$

$u = \cos(t)$
$du/dt = -\sin(t)$

THUS, $\mu(t) = \text{EXP}\left[\ln|\cos(t)|\right] \Rightarrow \mu(t) = \cos(t)$ FOR $0 \le t \le \pi/2$

(d) $\frac{dy}{dt} = 2(y-1)y$ CLEARLY, $y=0$ AND $y=1$ ARE EQUILIB. SOL'NS

$$\left.\frac{dy}{dt}\right|_{y=1/2} < 0 \qquad \left.\frac{dy}{dt}\right|_{y=-1} > 0 \qquad \left.\frac{dy}{dt}\right|_{y=2} > 0$$



(e) $y = u^{-1/2}$ $\quad \frac{dy}{dt} = -\frac{1}{2} u^{-3/2} \frac{du}{dt}$

PLUG INTO ORIGINAL ODE

$$-\frac{1}{2} u^{-3/2} \frac{du}{dt} - t \tan(t)\, u^{-1/2} = u^{-3/2}$$

DIVIDE THROUGH BY $u^{-3/2}$

$$-\frac{1}{2} \frac{du}{dt} - t \tan(t)\, u^{-1/2} u^{3/2} = 1$$

$$-\frac{1}{2} \frac{du}{dt} - t \tan(t)\, u = 1$$

$\boxed{3}$ LET $R(t)$ BE NUMBER OF RABBITS AT TIME t

MATH MODEL $\quad \frac{dR}{dt} = K\sqrt{R}$ FOR SOME CONSTANT K

$\boxed{4}$ (1) FIG B (2) FIG D (3) FIG C (4) FIG A

FOR EQ 3: $\frac{dy}{dt} = \frac{\cos y}{y+1}$ EQUILIBRIUM SOL'NS IN RANGE ARE $y = \pm \pi/2$

FROM DIR. FIELD, BOTH ARE STABLE

$\boxed{5}$