3. (a) F  (b) T  (c) F  (d) F

2. (a) (1) FIRST-ORDER, NONLINEAR, NONAUTONOMOUS

(2) SECOND-ORDER, LINEAR, HOMOGENEOUS, VARIABLE COEF., NONAUTONOMOUS

(b) \[
\frac{2y}{y^2+1} \frac{dy}{dt} = \frac{1}{t} \Rightarrow \int \frac{2y}{y^2+1} \, dy = \int \frac{1}{t} \, dt
\]

LHS: Let \( u = y^2 + 1 \) \\
\[
\frac{du}{dt} = 2y \frac{dy}{dt} \Rightarrow \int \frac{2y}{u} \, dy = \int \frac{1}{u} \, du = \ln |u| = \ln |y^2+1|
\]

Thus 
\[
\ln |y^2+1| = \ln |t| + C \quad y^2+1 = C \cdot t^2
\]
\[
y(1) = 1 \Rightarrow C = 2 \quad y = \sqrt{2t^2 - 1}
\]

Let \( f(y, t) = \frac{1+y^2}{2y} \) 

Clearly \( f(y, t) \) is cont. at \((1, 1)\)

\[
\frac{\partial f}{\partial y} = \frac{1}{2} \frac{\partial}{\partial y} \left( \frac{1}{y} + y \right) = \frac{1}{2} \left( y^2 + 1 \right) \frac{\partial f}{\partial y} \text{ is also cont at } (1, 1)
\]

By Picard's Thm, soln is unique about \((1, 1)\)

(c) STANDARD LINEAR FORM

\[
\frac{dy}{dt} - \frac{\sin(t)}{\cos(t)} y = \sin(t) \quad \mu(t) = \exp \left[ \int \frac{p(t)}{P} \, dt \right]
\]

\[
\int p(t) \, dt = \int \frac{-\sin(t)}{\cos(t)} \, dt = \int \frac{1}{u} \, du = \ln |u| = \ln |\cos(t)|
\]

\[
u = \cos(t) \quad \frac{du}{dt} = -\sin(t)
\]

Thus, \( \mu(t) = \exp \left[ \ln |\cos(t)| \right] \Rightarrow \mu(t) = \cos(t) \) for \( 0 \leq t \leq \frac{\pi}{2} \)

(d) \( \frac{dy}{dt} = 2(y-1) \) 

Clearly, \( y = 0 \) and \( y = 1 \) are equilibrium solns

\[
\left. \frac{dy}{dt} \right|_{y = \frac{1}{2}} < 0 
\]

\[
\left. \frac{dy}{dt} \right|_{y = 2} > 0 
\]

Phase line: 

- \( y = 1 \) (Unstable)
- \( y = 0 \) (Stable)
(e) \[ y = u^{-\frac{1}{2}} \]
\[ \frac{dy}{dt} = -\frac{1}{2} u^{-\frac{3}{2}} \frac{du}{dt} \]

Plug into original ODE

\[ -\frac{1}{2} u^{-\frac{3}{2}} \frac{du}{dt} = t \tan(t) u^{-\frac{1}{2}} = u^{-\frac{3}{2}} \]

Divide through by \( u^{-\frac{3}{2}} \)

\[ -\frac{1}{2} \frac{du}{dt} = t \tan(t) u^{-\frac{1}{2}} u^{\frac{3}{2}} = 1 \]

\[ -\frac{1}{2} \frac{du}{dt} = t \tan(t) u = 1 \]

3) Let \( R(t) \) be number of rabbits at time \( t \).

Math model \[ \frac{dR}{dt} = k \sqrt{R} \] for some constant \( k \).

4) (1) Fig B  (2) Fig D  (3) Fig C  (4) Fig A

For Eq 3: \[ \frac{dy}{dt} = \frac{\cos y}{y+1} \]
Equilibrium solutions in range \( y = \pm \frac{\pi}{2} \)

From Dir. field, both are stable.