1 (a) TRUE (b) FALSE (c) FALSE (d) TRUE (e) FALSE

2 (a) CHARACTERISTIC EQ'N: $r^2+2r+1=0 \Rightarrow (r+1)^2=0$

ROOTS: $r_1=r_2=-1$ $(\Delta=0) \Rightarrow$ $\boxed{\text{CRITICALLY DAMPED}}$

(b) $y_h(t)$ IS LINEARLY INDEP. W.R.T. $f(t)$: GUESS $y_p(t)=e^{-t}(At+B)$

$y_p'(t)= Ae^{-t}-Ate^{-t}-Be^{-t}$ $\quad y_p''(t)=-Ae^{-t}-Ae^{-t}+Ate^{-t}+Be^{-t}$

$(B-2A)e^{-t}+Ate^{-t}+4(A-B)e^{-t}-4Ate^{-t}+4e^{-t}(At+B)=te^{-t}$

t^0: $B-2A+4A-\cancel{4B}+\cancel{4B}=0 \Rightarrow B+2A=0 \Rightarrow B=-2$

t^1: $A-4A+4A=1 \Rightarrow A=1$

$$\boxed{y_p(t)=e^{-t}(t-2)}$$

(c) PLUG $y_p(t)=v_1y_1+v_2y_2$ INTO ODE, WITH RESTRICTION $v_1'y_1+v_2'y_2=0$

$y_p'(t)=v_1'y_1+v_1y_1'+v_2'y_2+v_2y_2' = v_1y_1'+v_2y_2'+\underbrace{v_1'y_1+v_2'y_2}_{=0} = v_1y_1'+v_2y_2'$

$y_p''(t)=v_1'y_1'+v_1y_1''+v_2'y_2'+v_2y_2''$

PLUG IN

$v_1'y_1'+v_1y_1''+v_2'y_2'+v_2y_2''+p(v_1y_1'+v_2y_2')+q(v_1y_1+v_2y_2)=f(t)$

$v_2\underbrace{(y_2''+py_2'+qy_2)}_{=0}+v_1\underbrace{(y_1''+py_1'+qy_1)}_{=0}+v_1'y_1'+v_2'y_2'=f(t)$

$$\boxed{\begin{aligned} v_1'y_1+v_2'y_2&=0\\ v_1'y_1'+v_2'y_2'&=f(t)\end{aligned}}$$

3 $\det(B)=K+2$; INVERTIBLE IF $\det B \neq 0$ $\quad \boxed{K\neq -2}$

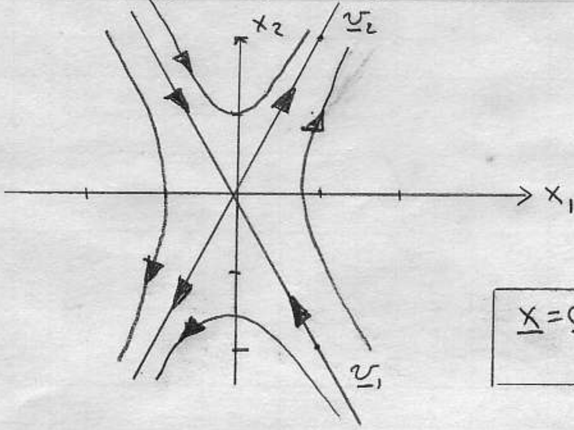
4 (a) $\lambda_{1,2}=\dfrac{tr(A)\pm\sqrt{tr(A)^2-4\det(A)}}{2} \Rightarrow \lambda_{1,2}=\frac{1}{2}\left(2\pm\sqrt{4-4(-3)}\right)=1\pm\sqrt{4}$

$\boxed{\lambda_1=-1,\ \lambda_2=3}$

$\underline{v}_1$: $(A-\lambda_1 I)\underline{v}_1=\underline{0} \Rightarrow \left[\begin{array}{cc|c}2&1&0\\4&2&0\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}2&1&0\\0&0&0\end{array}\right] \Rightarrow$ $\underline{v}_1=\begin{bmatrix}1\\-2\end{bmatrix}$

$\underline{v}_2$: $(A-\lambda_2 I)\underline{v}_2=\underline{0} \Rightarrow \left[\begin{array}{cc|c}-2&1&0\\4&-2&0\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}-2&1&0\\4&-2&0\end{array}\right] \Rightarrow$ $\underline{v}_2=\begin{bmatrix}1\\2\end{bmatrix}$

(b)



$\underline{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\lambda_1 = -1$

$\underline{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\lambda_2 = 3$

$\boxed{\underline{x} = \underline{0} \text{ IS AN UNSTABLE SADDLE NODE}}$

(c) ASSUME $\underline{x}(t) = \underline{v} e^{\lambda t}$ WHERE $\underline{v} \in \mathbb{C}^n, \lambda \in \mathbb{C}$

PLUG IN TO $\underline{x}' = A\underline{x}$

$\lambda \underline{v} e^{\lambda t} = A \underline{v} e^{\lambda t} \Rightarrow \boxed{A\underline{v} = \lambda \underline{v}}$

$\boxed{5}$ (1) $r^2 + r + 4 = 0 \Rightarrow r_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm i\sqrt{3}$ UNDERDAMPED

(2) $r^2 + 2r + 1 = 0 \Rightarrow r_1 = r_2 = 1$ CRITICALLY DAMPED

(3) $r^2 - r + 2 = 0$ $r_{1,2} = \frac{1 \pm \sqrt{1-8}}{2} = \frac{1}{2} \pm i\sqrt{7}$ UNSTABLE, OSCILLATORY

THUS

$\boxed{(1)\ C \quad (2)\ B \quad (3)\ E}$