Math 24

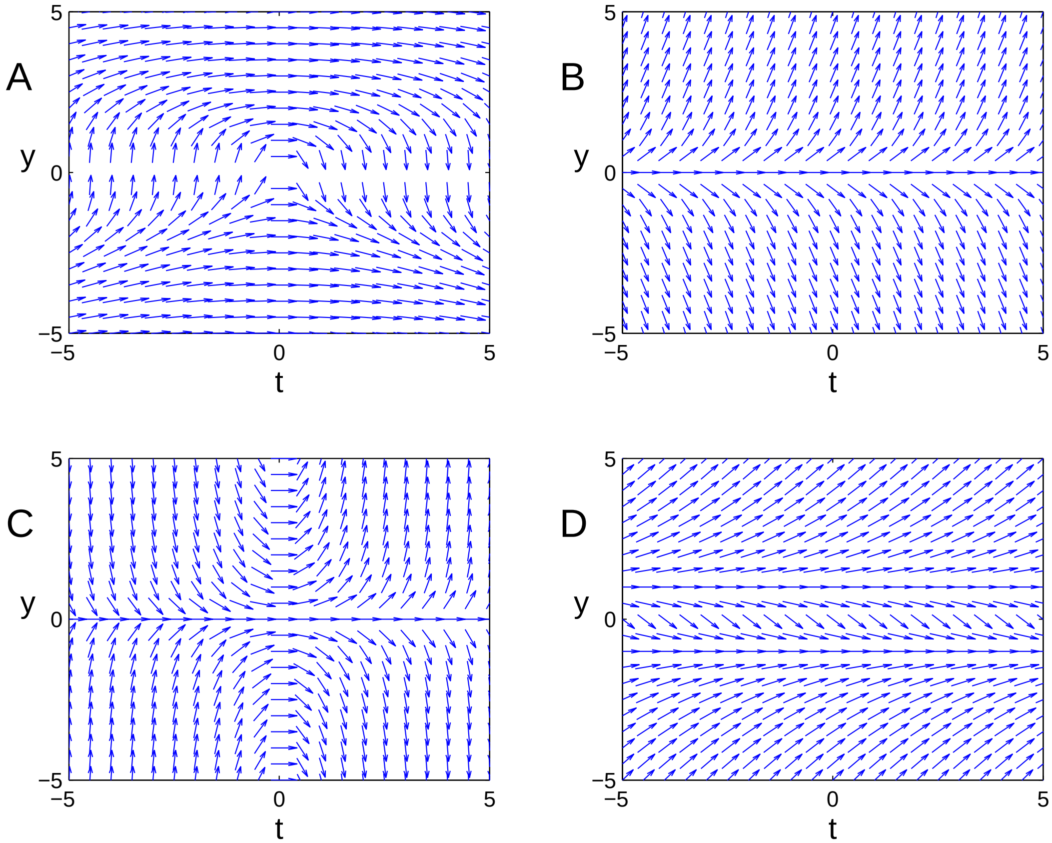
Exam 1: February 15, 2006

ON THE FRONT OF YOUR BLUEBOOK WRITE (1) YOUR NAME, (2) A FIVE-PROBLEM GRADING GRID. **Show ALL of your work** in your bluebook, and **box in your final answers.** A correct answer, but without the relevant work, will receive no credit. This exam is closed-book and no calculators are allowed. You are allowed a one-page crib sheet. Start each problem on **the top of a new page.** Each problem is worth 20 points, for a total of 100 points. You can solve the problems in any order you like.

1. (a) Match the following differential equations (1)–(4) with their corresponding direction fields A–D.

   (1) $\dfrac{dy}{dt} = y + \sin(y)$, (2) $\dfrac{dy}{dt} = +\sqrt{|y|} - 1$, (3) $\dfrac{dy}{dt} = -\dfrac{t^2}{y}$, (4) $\dfrac{dy}{dt} = ty$.

   (b) Do equations (1)–(4) have any equilibrium solutions? If so, find the equilibrium solutions for each equation.



2. Classify the following equations as (1) linear or nonlinear, and (2) separable or non-separable. Then solve these equations.

   (a) $\sin(t)y' + \cos(t)y = 1$.

   (b) $y' = \dfrac{t(t+1)}{y^2}$.

3. Consider the equation

$$y' = 2y^2(y^2 - a^2) \ ,$$

   where a is a positive constant ($a > 0$).

   (a) Find the equilibrium solutions.

   (b) Sketch the phase lines and direction fields.

   (c) Determine the stability of the equilibrium solutions.

   (d) Find the limiting value of $y(t)$ as $t \to \infty$ when $y(0) = \frac{a}{2}$. Justify your answer.

4. Determine whether the following statements are TRUE or FALSE. Note: you must write the entire word TRUE or FALSE. You do not need to show your work for this problem.

   (a) $y(x) = x\cos(x)$ is a solution of the IVP

$$y' - \frac{y}{x} = -x\sin(x) \ , \quad y(0) = 0 \ .$$

   (b) Picard's theorem guarantees the local existence and uniqueness of a solution to the IVP

$$\frac{dy}{dx} = \tan^{-1}(y) \ , \quad y(0) = c \ ,$$

   for all values of c.

5. Solve the equation

$$\frac{dy}{dt} - \frac{y}{t} = \left(\frac{y}{t}\right)^2 \ .$$

   Hint: since this is an *Euler-homogeneous equation*, you can use $v = \frac{y}{t}$ to transform it into a separable equation. Alternatively, since this is also a *Bernoulli equation*, you can use $v = y^{-1}$ to transform it into a linear equation.

THE END