1. Consider the linear system

\[
\begin{align*}
2x - 10z &= 0 \\
y + 2z &= 1 \\
-4x + 3y + 26z &= 3
\end{align*}
\]

(a) Write this system in the augmented form \([A|b]\).
(b) Using the determinant, determine whether the system has a unique solution.
(c) Solve this system using Gauss Elimination (RREF).
(d) What is the span of the solutions (point, line, plane, ...) and what is its dimension?

2. Let \(A = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & -1 \end{bmatrix}\) and \(b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\).

(a) Calculate \(AA^T\).
(b) Find \(A^{-1}\).
(c) Solve \(Ax = b\) when \(\theta = \frac{\pi}{2}\).
(d) Which of the following products are defined:
   (i) \(Ab\), (ii) \(bA\), (iii) \(A^Tb^T\), (iv) \(b^TA\), (v) \(b^Tb\)?
   Note: you do not need to calculate these products.

3. Consider the following system of equations in augmented form

\[
[A|b] = \begin{bmatrix} k & -2 & 1 \\ 2 & -k & -1 \end{bmatrix}
\]

(a) For which values of \(k\) does this system have a unique solution?
(b) Determine the number of solutions of this system for all values of \(k\).
(c) Define the properties of a basis of a vector space. Explain whether the column vectors of \(A\) form a basis of \(\mathbb{R}^2\) when (i) \(k = 0\) and (ii) \(k = 2\).
4. Determine whether the following statements are TRUE or FALSE. Note: you must write the entire word TRUE or FALSE. You do not need to show your work for this problem.

(a) The RREF of \( A = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 0 & 10 & 0 & 1 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & -3 \end{bmatrix} \) is the identity matrix.

(b) The vectors \( \{[-3, 1], [2, -3], [2, 1]\} \) span \( \mathbb{R}^2 \).

(c) The vectors \( \{[-3, 1], [2, -3], [2, 1]\} \) form a basis of \( \mathbb{R}^2 \).

(d) The functions \( \{t^2, t^2 + 1, t^2 + 3t + 1, t + 4\} \) are linearly independent.

(e) The set of solutions of the equation \( x^2y'' + xy' - y = 0 \) forms a vector space.

5. Consider the differential equation \( y'' + y' - 6y = 0 \).

(a) Find the characteristic equation and its roots.

(b) What is the general solution of this differential equation?

(c) Solve the equation when \( y(0) = 0 \) and \( y'(0) = 5 \).

(d) What is the long time behavior of the solution?