**MATH 24 – EXAM 3** **2:00-3:30pm May 3rd, 2006**

ON THE FRONT OF YOUR BLUEBOOK WRITE (1) YOUR NAME, (2) A FIVE-PROBLEM GRADING GRID. **Show ALL of your work** in your bluebook, and **box in your final answers.** A correct answer, but without the relevant work, will receive no credit. This exam is closed-book and no calculators are allowed. You are allowed a one-page crib sheet. Start each problem on **the top of a new page.** Each problem is worth 20 points, for a total of 100 points. You can solve the problems in any order you like.

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1. Consider the equation

$$y'' + \omega_0^2 y = \cos \omega_1 t \ .$$

(a) Find the general solution of the homogeneous equation.

(b) Find the particular solution of the non-homogeneous equation using the method of <u>Undetermined Coefficients</u> assuming that $\omega_1 \neq \omega_0$.

(c) What is the suitable guess for the particular solution if $\omega_1 = \omega_0$?

(d) Suppose $\omega_1 > \omega_0$, but as time evolves, $\omega_1$ is getting closer and closer to $\omega_0$. What happens to the qualitative behavior of the solution in this process? Is this change of behavior consistent with your answer to part (c)?

2. Consider the equation

$$y'' - 2y' + y = \frac{e^t}{t} \ .$$

(a) Find the general solution of the homogeneous equation.

(b) Find the Wronskian of the fundamental solutions of the homogeneous equation.

(c) Find the particular solution of the non-homogeneous equation using the method of <u>Variation of Parameters</u>.

(d) Convert this equation to a system of first-order equations and write it using vector-matrix notation.

3. Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \ .$$

(a) Find the eigenvalues.

(b) Find the eigenvectors.

(c) What is the dimension of the span of all the eigenvectors?

(d) Does the algebraic system $A\mathbf{x} = \mathbf{0}$ have a unique solution?

4. Determine whether the following statements are TRUE or FALSE. Note: you must write the entire word TRUE or FALSE. You do not need to show your work for this problem.

   (a) Let $y'' - 2y' + y = e^t \cos t$. Then $y_p = e^t(A\cos t + B\sin t)$ is a suitable guess for the particular solution.

   (b) Let $y'' + y' = t^2 + 1$. Then $y_p = At^2 + Bt + C$ is a suitable guess for the particular solution.

   (c) Suppose the eigenvalues of a matrix are $\{-1, 0, 1\}$. Then the matrix is invertible.

   (d) Let
$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 7 \\ -1 & 0 & 2 \end{bmatrix}.$$
   Then the sum of its eigenvalues is 3.

   (e) The eigenvalues of the fourth-order equation $y'''' - y = 0$ are distinct (i.e., different from each other).

5. Consider the system of differential equations $\mathbf{x}' = A\mathbf{x}$ with $A = \begin{bmatrix} -3 & 2 \\ -5 & 3 \end{bmatrix}$.

   (a) Find the eigenvalues.

   (b) Find the eigenvectors.

   (c) Find the general solution.

   (d) Solve the initial value problem with $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

   (e) What is the stability structure of the equilibrium solution?

THE END