MATH 24 – Midterm 2 *Fall Semester 2007*

Duration: 50 minutes
Instructions: Answer all questions, without the use of notes, books or calculators. Partial credit will be awarded for correct work, unless otherwise specified. The total number of points is 50.

1. (12 pts) Find the general solution to $x'' - 2x' + 10x = -2e^{-t}$.

2. (8 pts) A system of two equations of two unknown functions has the origin as an equilibrium point. Using the following eigenvalues and eigenvectors, sketch approximate phase portraits of the solutions and determine the stability of the origin.

   a) $\lambda_1 = 2$ with $\vec{v}_1 = (1, -1)^T$, and $\lambda_2 = -1$, with $\vec{v}_2 = (1/4, 2)^T$.

   b) $\lambda_1 = \lambda_2 = -3$ with $\vec{v}_1 = (-3, 2)^T$, and $\vec{v}_2 = (-1, -2)^T$.

3. (12 pts) Find all the eigenvalues of the matrix A below. Choose a specific eigenvalue and find its corresponding eigenspace.

$$A = \begin{bmatrix} 2 & 2 & -4 \\ 0 & 4 & 0 \\ 4 & 1 & 2 \end{bmatrix}$$

4. (8 pts) Find the solution of the initial value problem given below for which $x_1(0) = 10$ and $x_2(0) = 0$, using the fact that the eigenvalues of the corresponding matrix are $\lambda_1 = 2$ and $\lambda_2 = -3$.

$$\begin{aligned} \frac{dx_1}{dt} &= -4x_1 - x_2 \\ \frac{dx_2}{dt} &= 6x_1 + 3x_2 \end{aligned}$$

5. (10 pts) Answer the following questions in no more than two lines of text (much less is actually needed if you are right on point). Minimal (if any) computation is required.

   a) What initial conditions are satisfied at $t = 0$ by $y(t) = 2\sin 2t - 3\cos 2t$, a solution to $y'' + 4y = 0$?

   b) What happens to the amplitude of the particular solution of a harmonic oscillator which is forced at its natural oscillation frequency? What is this phenomenon called?

   c) In the method of variation of parameters, we guess that the particular solution is $y_p(t) = v_1 y_1 + v_2 y_2$. What are the functions y_1 and y_2 if v_1 and v_2 are unknown functions?

   d) Sketch the image of the transformation $T(\vec{x}) = \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x, -x)$.

   e) If $y(t) = t^3$ is a solution of $t^2 y'' + ty' - 9y = 0$, argue that $y(t) = t^3/7$ is also a solution.