Math 24

Exam 2: March 21, 2007

ON THE FRONT OF YOUR BLUEBOOK WRITE (1) YOUR NAME, (2) A **THREE**-PROBLEM GRADING GRID. **Show ALL of your work** in your bluebook. A correct answer, but without the relevant work, will receive no credit. You are allowed a one-page crib sheet. Each problem is worth 30 points for a total of 90 points.

1. Answer the following TRUE/FALSE questions (write only the word TRUE or FALSE):
   (a) If $A\mathbf{x} = \mathbf{0}$ has two different solutions, $\mathbf{x}_1 \neq \mathbf{x}_2$, then A has more rows than columns.
   (b) The functions $\{1, \sin^{-1} t, \cos^{-1} t\}$ are linearly dependent.
   (c) The span of $\{1, 1+t, 1-t^2, 2t+t^2\}$ is $\mathbb{P}^2$.
   (d) The vectors $\{[1,0,1], [0,4,0], [1,5,2], [3,-1,5]\}$ form a basis for $\mathbb{R}^3$.
   (e) The set of solutions to the differential equation $\frac{1}{t^2}y'' - \frac{1}{t}y' = \frac{1}{t^2}$ forms a two-dimensional vector space.
   (f) The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 5 & 5 & -5 \end{bmatrix}$ commutes with its inverse.

2. Consider the linear system of equations

$$\begin{aligned} x + 3z &= 1 \\ -4x + 5y - 17z &= 1 \\ y - z &= 1 \end{aligned}$$

   (a) Write this system in the augmented form $[A|\mathbf{b}]$.
   (b) Using the determinant, determine whether the system has a unique solution.
   (c) Solve this system using Gauss Elimination (RREF).
   (d) What do the solutions span geometrically (a point, line, plane, space, etc) and what is the dimension of this span?
   (e) <u>Define</u> a basis for a vector space (in general); and determine whether the solutions of the system above form a basis for $\mathbb{R}^3$.

3. Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.
   (a) Calculate the determinant of A.
   (b) Show that $A^{-1} = A^T$.
   (c) Solve $A\mathbf{x} = \mathbf{b}$.
   (d) What is the RREF of A?
   (e) Which of the following are defined: (i) $\mathbf{b}A$, (ii) $A^T\mathbf{b}^T$, (iii) $A\mathbf{b}$, (iv) $\mathbf{b}^TA$, (v) $\mathbf{b}^T\mathbf{b}$? Note: you do not need to calculate these products.
   (f) Find an example of three vectors or three functions that are linearly *dependent*, but such that each two of them are linearly *independent*.

THE END