Math 24
Exam 3: April 25, 2007

ON THE FRONT OF YOUR BLUEBOOK WRITE (1) YOUR NAME, (2) A **THREE**-PROBLEM GRADING GRID. **Show ALL of your work and EXPLAIN your answers** in your bluebook. A correct answer, but without explanation, will receive no credit. You are allowed a one-page crib sheet. Each problem is worth 36 points for a total of 108 points.

1. Consider the following second-order differential equation

$$y'' - 4y' + 4y = \frac{e^{2t}}{1 + t^2} \ .$$

   (a) Find the general solution of the homogeneous equation.

   (b) Find a particular solution of the non-homogeneous equation using the method of Variation of Parameters.

   (c) Convert this equation to a first-order system, that is, write it in the form $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$.

   (d) What are the eigenvalues of the matrix A in part (c)?

2. Consider the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 0 & 6 & -5 \end{bmatrix}$

   (a) Find the eigenvalues of A (Explain).

   (b) Find the eigenvectors of A.

   (c) Find the general solution of the system $\mathbf{x}' = A\mathbf{x}$.

3. Write down a suitable guess for the particular solution using the method of Undetermined Coefficients (you do NOT need to calculate the coefficients, but SHOW your work):

   (a) $y'' - 2y' = 1 + t$

   (b) $y'' - 2y' + y = e^t$

   (c) $y'' - 2y' + y = te^t$

   (d) $y'' + 4y = \cos 2t$

THE END