FINAL EXAM, MATH 24, FALL 2008

Instructions: Do not begin the exam until you are instructed to do so. You may write on the exam sheet, but ONLY what is written in your bluebook will be graded.

For each problem, you must show all work in order to receive credit. Partial credit will be given when appropriate, even if the final answer is not correct, but an answer with no work shown will receive zero credit regardless of correctness. You may not use any text, notes, or calculators on this exam, and collaboration is not allowed.

1. (10 points) Solve the IVP

$$\begin{aligned} y' + ty &= t \\ y(0) &= 3 \end{aligned}$$

2. For the inhomogeneous system

$$\begin{aligned} x' &= 4x - y + 6 \\ y' &= 2x + y \end{aligned}$$

   a. (10 points) Find the equilibrium solution(s).

   b. (5 points) Write the system in matrix-vector form.

3. (10 points) What value or values of k make the vectors $\begin{bmatrix} 1 \\ 2 \\ k \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} k \\ -1 \\ 3 \end{bmatrix}$ linearly dependent?

4. For the matrix $\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$,

   a. (10 points) Find the eigenvalues.

   b. (10 points) Find two linearly independent eigenvectors.

5. (10 points) Find the general solution to the differential equation

$$y'' - 2y' - 2y = 2t$$

!! MORE ON THE BACK !!

6. (2 points) Find the Laplace Transform for the function $f(t) = \sinh(t)$, <u>for $s > 1$</u>. HINT: Express the function in terms of exponentials.

7. The following statements are incorrect, or at least not *entirely* accurate. Explain why with complete sentences and/or an example.

a. (3 points) The column vectors of any square matrix form a basis for the column space of that matrix.

b. (3 points) Any plane in the vector space $\mathbb{R}^3$ can be specifically considered as the vector space $\mathbb{R}^2$, so all planes in $\mathbb{R}^3$ are vector subspaces.

c. (3 points) The linear system $A\mathbf{x} = \mathbf{b}$ will have infinitely many solutions if A is a singular matrix.

d. (3 points) A repeated characteristic root of a second-order differential equation can be either real or complex.

e. (3 points) If a 2nd-order linear system of (first-order) DE's has a repeated eigenvalue, then the eigenvectors cannot possibly span the eigenspace, and you must use a generalized eigenvector (i.e. $\mathbf{u}$ such that $(A - \lambda I)\mathbf{u} = \mathbf{v}$, where λ is an eigenvalue, and $\mathbf{v}$ is its eigenvector) to obtain 2 linearly independent eigenvectors.

f. (3 points) If a matrix transforms a vector such that the resulting vector remains in the same vector space as the original, then the original vector is an eigenvector of the transformation matrix.

8. A tank contains 100 gallons of pure water. At time $t = 0$, saltwater containing 5 pounds of salt per gallon is pumped in at a rate of 2 gallons per minute. The mixture is drained such that the volume of liquid in the tank remains constant.

a. (10 points) Find an expression for the amount of salt in the tank at time t.

b. (5 points) As $t \to \infty$, how much salt is in the tank?