

MATH 24 – Exam 1

Spring Semester 2008

Instructions. Please provide clear and concise solutions to the problems in this exam in your bluebook. Where applicable, explain your reasoning using complete sentences with proper grammar. Poor presentation may result in loss of credit.

1. (10 pts) Find the general solution of

$$\frac{dy}{dt} + \frac{y}{t} = e^{-t}, \quad t > 0.$$

2. For each of the two items below, please write a clear and concise written response.

- (a) (5 pts) Explain why you would be able to determine that a first-order differential equation is *autonomous* solely from its direction field.
- (b) (5 pts) Explain why we cannot use Euler's method to compute the *general solution* of a first-order differential equation.

3. (10 pts) Solve the initial value problem:

$$\frac{dy}{dt} = \frac{2t}{y + t^2y}, \quad y(0) = -2.$$

4. (10 pts) A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ grams/liter of salt enters the tank at a rate of 2 liters/minute, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t . Also, find the limiting amount of salt in the tank as $t \rightarrow \infty$.

5. (10 pts) Explain what Picard's theorem tells us about the initial value problem below?

$$\frac{dy}{dt} = \frac{1}{y - t^2}, \quad y(0) = 0.$$