# MATH 24 – Exam 3

Spring Semester 2008

**Instructions.** Please provide clear and concise solutions to the problems in this exam in your bluebook. Where applicable, explain your reasoning using complete sentences with proper grammar. Poor presentation may result in loss of credit.

1. (10 points) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}.$$

2. For the following initial-value problem,

$$\frac{\mathrm{d}^2y}{\mathrm{d}t} + \frac{\mathrm{d}y}{\mathrm{d}t} - 6y = 6te^{-t}, \quad y(0) = \frac{1}{6}, \quad y'(0) = -\frac{1}{6}.$$

   (a) (5 points) Find the homogeneous solution.

   (b) (5 points) Find the particular solution using the method of undetermined coefficients.

   (c) (5 points) Find the general solution of the differential equation.

   (d) (5 points) Find the unique solution of the initial-value problem.

3. For the differential equation

$$y'' + \omega^2 y = f(t), \quad (\omega \text{ is a positive, real constant})$$

   (a) (5 points) Verify that $y_1(t) = \cos(\omega t)$ and $y_2(t) = \sin(\omega t)$ are solutions of the homogeneous problem.

   (b) (5 points) Compute the Wronskian $W[y_1, y_2](t)$ and show that the set of homogeneous solutions is linearly independent as long as $\omega \neq 0$.

   (c) (5 points) Use variation of parameters to find the particular solution in terms of integrals involving $f(t)$.

   (d) (5 points) Combine all of your results and show that the general solution of the nonhomogeneous problem is given by

$$y(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t) + \frac{1}{\omega} \int_0^t \sin[\omega(t - \tau)] f(\tau) \mathrm{d}\tau.$$

   Here, you will need to use the fact that $\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$.