Math 24
Exam 1: February 23, 2009

ON THE FRONT OF YOUR BLUEBOOK WRITE (1) YOUR NAME, (2) A **FOUR**-PROBLEM GRADING GRID. **Show ALL of your work** in your greenbook/bluebook and **box in your final answers.** Unless otherwise mentioned, an answer without the relevant work will receive no credit. Start each problem at **the top of a new page.** You can solve the problems in any order you like.

1. (30 points) Consider the equation $\dfrac{dy}{dt} = ty + ty^2$.
   (a) Define what an equilibrium solution is in general.
   (b) Find the equilibrium solutions of the equation above.
   (c) Sketch the direction field and determine the stability of the equilibrium solutions.
   (d) Find the general solution. What is its long-time behavior?
   (e) Perform the transformation $v(t) = \dfrac{1}{y(t)}$ and obtain a differential equation for $v(t)$. Classify this equation (you do not need to solve it).

2. (30 points) Consider the equation $\dfrac{dy}{dt} + \dfrac{y}{\sqrt{t}} = e^{-\sqrt{t}}$.
   (a) Classify this equation as best you can.
   (b) Explain what Picard's theorem says about the existence and uniqueness of a solution with the initial value $y(0) = 1$.
   (c) Find the general solution.
   (d) Find the solution with initial value $y(0) = 1$. Is your answer consistent with part (b)?

3. (20 points) Answer the following TRUE/FALSE questions. Explain your answers concisely and be sure to write the word TRUE or FALSE.
   (a) $y(x) = x \sin x$ is a solution of the IVP $\dfrac{dy}{dx} - \dfrac{y}{x} = -x\cos x\ , \quad y(0) = 0$.
   (b) $y = t$ is an equilibrium solution of $\dfrac{dy}{dt} = t\left[\tan(y) - \tan(t)\right]$.
   (c) Picard's theorem guarantees existence and uniqueness of a solution to the IVP

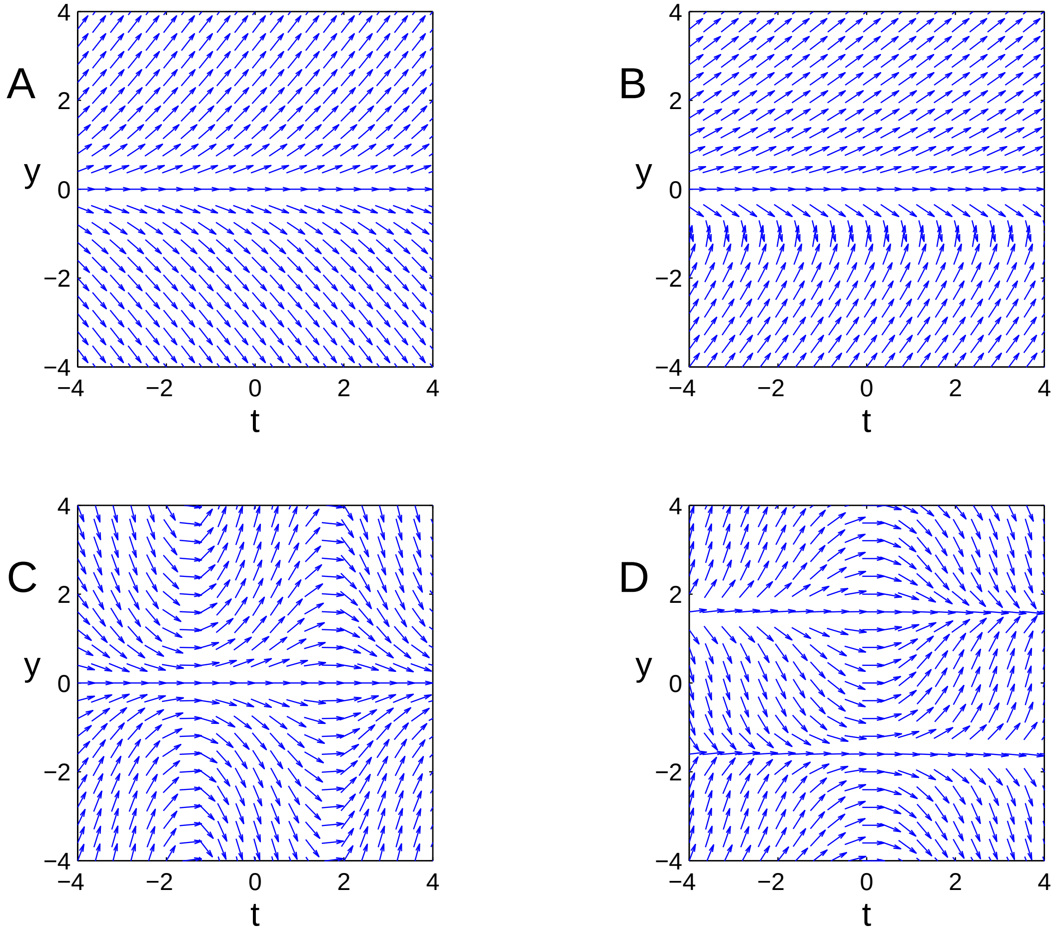
   $$\frac{dy}{dx} = \sin^{-1} y\ , \quad y(0) = 0\ .$$

   (d) If $y_1(t)$ and $y_2(t)$ are two solutions of the equation $\sin(t)\dfrac{dy}{dt} - ty = 0$ then $y_1 + y_2$ is also a solution of this equation.

TURN OVER

4. (20 points) Match the following equations (1)–(4) with their corresponding direction fields A–D (you do not need to show your work for this problem):

$$(1)\ \frac{dy}{dt} = t\cos y\,, \quad (2)\ \frac{dy}{dt} = y\cos t\,, \quad (3)\ \frac{dy}{dt} = \frac{y}{1+y}\,, \quad (4)\ \frac{dy}{dt} = \tan^{-1} y\,.$$



THE END