MATH 24, Exam 2, March 20, 2009

ON THE FRONT OF YOUR BOOK WRITE (1) YOUR NAME, (2) A **THREE**-PROBLEM GRADING GRID. **Show ALL of your work** and **box in your final answers.** Unless otherwise mentioned, an answer without the relevant work will receive no credit. You may solve the problems and each part of a problem in any order you like.

1. (33 points) Answer the following TRUE/FALSE questions. You must write the entire word TRUE or FALSE.
   (a) The vectors $\{[1,0,1],[2,0,2],[0,1,0]\}$ are linearly independent.
   (b) The dimension of the span of $\{[1,0,1],[2,0,2],[0,1,0]\}$ is 3.
   (c) The functions $\{1, 1+t, 2+t\}$ are linearly independent.
   (d) If $A\mathbf{x} = \mathbf{0}$ has a solution $\mathbf{x} \neq \mathbf{0}$ then $A\mathbf{x} = \mathbf{b}$ must have infinitely many solutions for any $\mathbf{b}$.
   (e) Four vectors in $\mathbb{R}^3$ are necessarily linearly dependent.
   (f) Four vectors in $\mathbb{R}^3$ necessarily span $\mathbb{R}^3$.
   (g) The set of solutions of the differential equation $y' + ty = t$ forms a vector space.

2. (32 points) Consider the linear system of equations

$$\begin{aligned} -x + y - z &= 42 \\ x + y + z &= 0 \end{aligned}$$

   (a) Find the solutions of this system using Gauss Elimination.
   (b) What do the solutions of the above system span geometrically (a point, line, plane, ...)?
   (c) What is the definition in general of a *basis* of a vector space?
   (d) Determine whether the solutions of the system above form a basis for $\mathbb{R}^3$.

3. (30 points) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & k & -1 \\ 0 & 1 & k \end{bmatrix}$ where k is a real constant, and let $\mathbf{b} = \begin{bmatrix} 42 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.
   (a) Calculate the determinant of A. For which values of k does A^{-1} exist?
   (b) Find A^{-1} when $k = 0$.
   (c) Find the solutions of $A\mathbf{x} = \mathbf{0}$ when $k = 0$.
   (d) Which of the following operations are defined (short answers): (i)$A - 9I$, (ii) $|A\mathbf{b}|$, (iii) $\mathbf{b}^T + \mathbf{b}$, (iv) $(\mathbf{b}^T A)^T$, (v) $\mathbf{b}^T\mathbf{b}$, (vi) $\mathbf{b}\mathbf{b}^T$?
   (e) Find an example of two 2x2 matrices, A and B, such that $AB \neq BA$.

HAPPY SPRING BREAK!