MATH 24, Exam 3, April 27, 2009

ON THE FRONT OF YOUR BOOK WRITE (1) YOUR NAME, (2) A **THREE**-PROBLEM GRADING GRID. **Show ALL of your work and box in your final answers.** Unless otherwise mentioned, an answer without the relevant work will receive no credit. You may solve the problems and each part of a problem in any order you like.

1. (32 points) Consider the equation $y'' - 9y = 9e^{-3t}$.
   (a) Find the general solution of the homogeneous equation.
   (b) Find the particular solution of the non-homogeneous equation.
   (c) What is the general solution of the non-homogeneous equation and what is its long time behavior?
   (d) Find the solution of the equation that satisfies $y(0) = 0,\ y'(0) = -\frac{3}{2}$.

2. (32 points)
   (a) Define (in general) the eigenvalues and eigenvectors of a matrix.
   (b) Find (and justify your answer) the eigenvalues of $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
   (c) Find the eigenvectors of A.
   (d) What is the dimension of the subspace spanned by all the eigenvectors of A?

3. (35 points) Answer the following TRUE/FALSE questions. You must write the entire word TRUE or FALSE. You do NOT need to justify your answers.
   (a) The equation $y'' - ky = \cos(\sqrt{2}\,t)$ corresponds to a resonant system if $k = 2$.
   (b) $y_p = Ate^{-3t}$ is a suitable guess for the particular solution of $y'' + 6y' + 9y = 2e^{-3t}$.
   (c) $y_p = A\cos(t^2) + B\sin(t^2)$ is a suitable guess for the particular solution of $y'' - y = \cos(t^2)$.
   (d) The system $\{x_1' = x_2,\ x_2' = 2x_2 - 2x_1\}$ has the same eigenvalues as $y'' - 2y' + 2y = 0$.
   (e) Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. Then A is invertible and the eigenvalues of A^{-1} are $\frac{1}{2}$ and $\frac{1}{3}$.
   (f) If A is a 2×2 matrix with a complex eigenvalue then A is invertible.
   (g) If A is any 2×2 matrix and c is any number then the eigenvalues of cA are c times the eigenvalues of A.

THE END