Math 30: Unit 3 Exam
Fall Semester 2006

1. (8 points) You model the population P of cells growing in culture as a function of time by the equation

$$\frac{dP}{dt} = 0.1P.$$

At time $t = 1$, we count 100 cells. Give a formula for the number of cells at $t = 0$.

2. (8 points) For a function $f(x)$, its two-term Taylor approximation about $x = x_0$ is given by

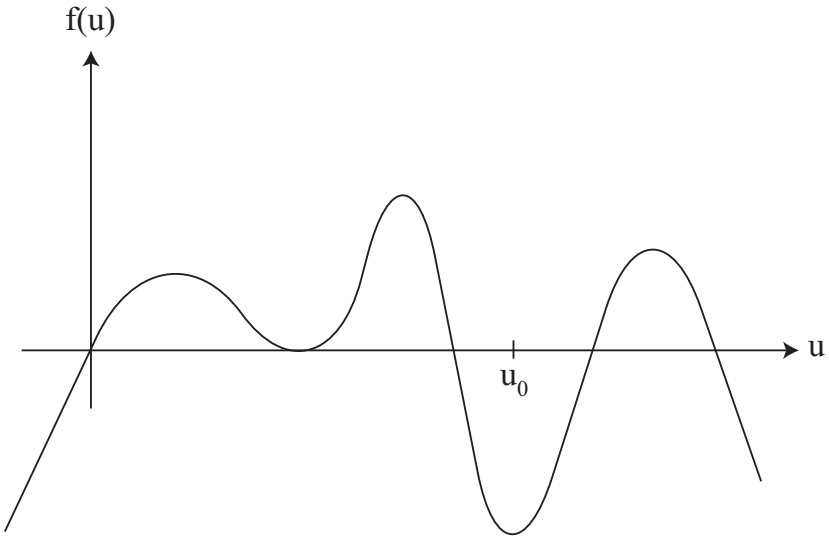
$$f(x) \approx f(x_0) + (x - x_0)f'(x_0) + \frac{1}{2!}f''(x_0) + \frac{1}{3!}f'''(x_0) + \cdots .$$

Find the two non-zero terms of the Taylor approximation for $f(x) = e^{-x}$ about $x = 0$.

3. (8 points) Consider the differential equation

$$\frac{du}{dt} = f(u)$$

with the graph of $f(u)$ given below.



Analyze this differential equation graphically. Identify, label and classify all of its equilibrium points directly on the graph above. Using that information, describe the long-time behavior of the solution when $u(0) = u_0$.