Midterm 1: Math 30, 9/20/07

1) A region is formed by $y = \frac{1}{4}x^2$ and y=x

a) Sketch the region
b) Find the area between two curves.
c) If the region is rotated about the x axis find the volume of the solid

2) A region is formed by $y = 6x - x^2$ and $x = 0$

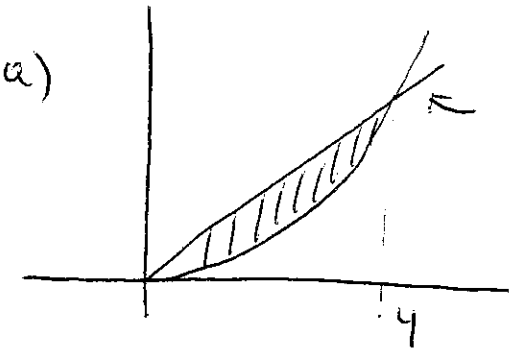
a) Sketch the region
b) Find the area of the region
c) If the region is rotated about the x-axis, find the volume of the solid
d) If the region is rotated about the y=axis, find the volume of the solid

3) A spring with constant k = 5 N/m and rest length of .05 m is stretched additional .025 m. Find the work done. $F=k(x-x_0)$

4) Temperature varies as $T(t) = 10 + 5\sin(\frac{\pi}{24}t)$ during the day where t is in hours. What is the average temperature during 24 hours?

Midterm 1: SEPTEMBER 21, 2007

1) a)

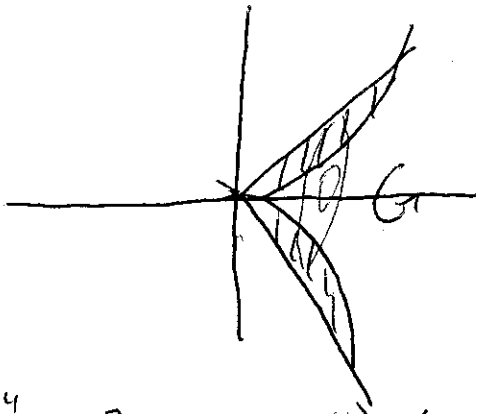


$\frac{1}{4}x^2 = x$ $x = 4$ intersection

b) $$A = \int_0^4 \left(x - \frac{1}{4}x^2\right) dx$$

$$= \frac{x^2}{2} - \frac{x^3}{12}\Big|_0^4 = \boxed{8/3}$$

c)



use the washer method

$dv = (\pi r_1^2 - \pi r_2^2)dy$

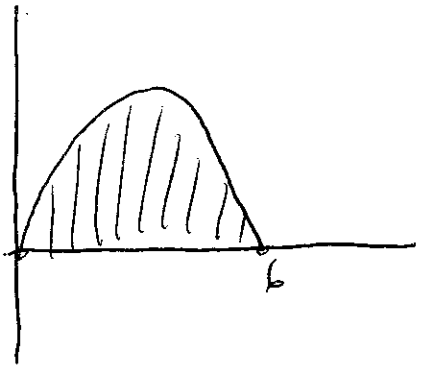
$$V = \int_0^4 \left(\pi x^2 - \frac{\pi x^4}{16}\right) dx$$

$$V = \frac{\pi x^3}{3} - \frac{\pi x^5}{80}\Big|_0^4 =$$

$$= \boxed{\pi\left(\frac{4^3}{3} - \frac{4^5}{80}\right) = \frac{128\pi}{15}}$$

(2)

a)

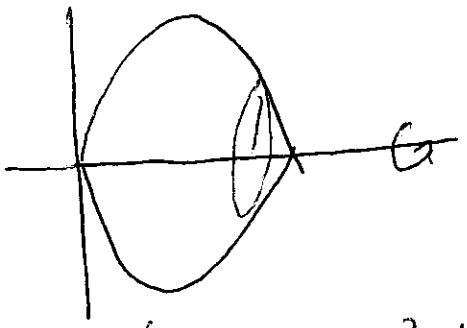


$y = 6x - x^2$

b) $A = \int_0^6 (6x - x^2)\,dx$

$= \frac{6x^2}{2} - \frac{x^3}{3}\Big|_0^6 = 6^3\left(\frac{1}{6}\right) = 36$

c)



$dV = \pi r^2 dx$

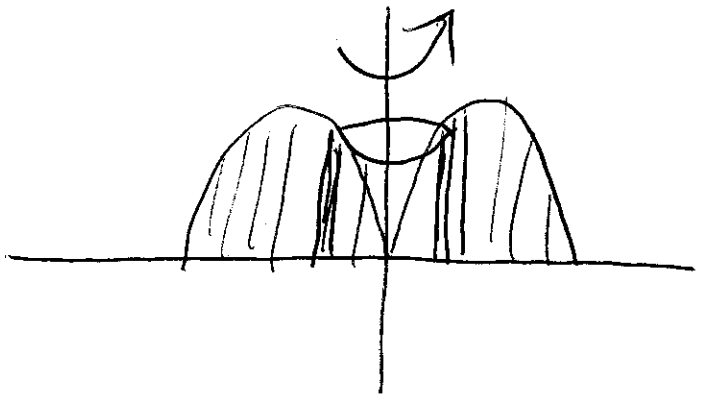
$V = \int_0^6 \pi (6x - x^2)^2\,dx$

$V = \pi \int_0^6 (36x^2 - 12x^3 + x^4)\,dx$

$V = \pi\left(\frac{36x^3}{3} - \frac{12x^4}{4} + \frac{x^5}{5}\right)\Big|_0^6$

$V = \pi 6^3\left(12 - 3 \cdot 6 + \frac{36}{5}\right) = \frac{1296}{5}\pi$

d)



use the shell method

$$dV = 2\pi r h\, dx$$

$$V = \int_0^6 2\pi x(6x - x^2)\, dx$$

$$V = 2\pi \int_0^6 6x^2 - x^3\, dx$$

$$= 2\pi \left(\frac{6x^3}{3} - \frac{x^4}{4}\right)\Big|_0^6$$

$$= 2\pi\, 6^4 \left(\frac{1}{12}\right) = 6^3\pi = \boxed{216\pi}$$

(3) $$W = \int F(x)\,dx = 5\int_{.05}^{.075} (x - .05)\,dx$$

$$F = 5(x - .05)$$

$$W = 5\left(\frac{x^2}{2} - .05x\right)\Big|_{.05}^{.075}$$

$$= \boxed{5\left(\frac{.075^2}{2} - (.05)(.075)\right) - 5\left(\frac{.05^2}{2} - .05^2\right)}$$

$$= \boxed{1.56 \times 10^{-3}\ \text{J}}$$

(4) $T_{AV} = \frac{1}{24}\int_0^{24} 10 + 5\sin\left(\frac{\pi}{24}t\right)dt$

$$T_{AV} = \frac{1}{24}\left(10t - \frac{120}{\pi}\cos\left(\frac{\pi t}{24}\right)\right)\Bigg|_0^{24}$$

$$T_A = \left(10 + \frac{5}{\pi}\right) - \left(-\frac{5}{\pi}\right) = \boxed{10 + \frac{10}{\pi}}$$