1) A region is formed by \( y = \frac{1}{4}x^2 \) and \( y = x \)

a) Sketch the region 
b) Find the area between two curves. 
c) If the region is rotated about the x axis find the volume of the solid

2) A region is formed by \( y = 6x - x^2 \) and \( x = 0 \)

a) Sketch the region 
b) Find the area of the region 
c) If the region is rotated about the x-axis, find the volume of the solid 
d) If the region is rotated about the y-axis, find the volume of the solid

3) A spring with constant \( k = 5 \) N/m and rest length of .05 m is stretched additional .025 m. Find the work done. \( F = k(x-x_0) \)

4) Temperature varies as \( T(t) = 10 + 5\sin\left(\frac{\pi}{24}t\right) \) during the day where \( t \) is in hours. What is the average temperature during 24 hours?
Midterm 1: SEPTEMBER 21, 2007

1) a) 
\[ r = \frac{1}{4} x^2 = x \quad x = 4 \text{ intersection} \]

\[ y \]

b) \[
A = \int_0^4 (x - \frac{1}{4} x^2) \, dx \\
= \frac{x^2}{2} - \frac{x^3}{12} \bigg|_0^4 = \frac{8}{3}
\]

c) \[
V = \int_0^4 (\frac{\pi x^2}{16} - \frac{\pi x^4}{16}) \, dx \\
= \frac{\pi x^3}{3} - \frac{\pi x^5}{80} \bigg|_0^4 = \pi \left( \frac{4^3}{3} - \frac{4^5}{80} \right) \\
= \frac{128\pi}{15}
\]

use the washer method
\[
dV = (\pi r_1^2 - \pi r_2^2) \, dy
\]
a) \[ y = 6x - x^2 \]

b) \[ A = \int_0^6 (6x - x^2) \, dx \]
\[ = \left. \frac{6x^2}{2} - \frac{x^3}{3} \right|_0^6 \]
\[ = 6^3 \left( \frac{1}{6} \right) = 36 \]

c) \[ dV = \pi x^2 \, dx \]
\[ V = \int_0^6 \pi (6x - x^2)^2 \, dx \]
\[ V = \pi \int_0^6 (36x^2 - 12x^3 + x^4) \, dx \]
\[ V = \pi \left( \frac{36x^3}{3} - \frac{12x^4}{4} + \frac{x^5}{5} \right) \bigg|_0^6 \]
\[ V = \pi 6^3 \left( 12 - 3 \cdot 6 + \frac{36}{5} \right) = \frac{1296 \pi}{5} \]
Use the shell method
\[ dv = 2\pi rh \, dy \]
\[ V = \int_0^b 2\pi x (6x - x^2) \, dy \]
\[ V = 2\pi \int_0^b (6x^2 - x^3) \, dy \]
\[ = 2\pi \left[ \frac{6}{3}x^3 - \frac{1}{4}x^4 \right]_0^b \]
\[ = 2\pi \frac{1}{4}b^4 \left( \frac{1}{12} \right) = \frac{6^3\pi}{216}\pi = \boxed{216\pi} \]

\[ W = \int F(x) \, dx = 5 \int_{0.05}^{0.075} (x - 0.05) \, dx \]
\[ F = 5(x - 0.05) \]
\[ W = 5 \left[ \frac{x^2}{2} - 0.05x \right]_{0.05}^{0.075} \]
\[ = \frac{5}{2} \left( 0.075^2 - (0.05 \times 0.075) \right) - \frac{5}{2} (0.05^2 - 0.05^2) \]
\[ = \boxed{1.56 \times 10^{-3}} \]
\[ T_{AV} = \frac{1}{24} \int_0^{24} \left( 10 + \frac{5}{24} \sin \left( \frac{\pi t}{24} \right) \right) dt \]

\[ T_{AV} = \frac{1}{24} \left. \left( 10t - \frac{120}{\pi} \sin \left( \frac{\pi t}{24} \right) \right) \right|_0^{24} \]

\[ T_A = \left( 10 + \frac{5}{\pi} \right) - \left( -\frac{5}{\pi} \right) = \frac{10 + 10}{\pi} \]