Midterm 3: Math 30, 11/19/07

5 pts 1) Find the length of the curve. Give the final number for full credit.

\[ y = \left(4 - x^2\right)^{1/2}, 0 < x < 1 \]

5 pts 2) Find the area of the surface obtained by rotating the curve about the \( x \)-axis.

\[ x = \frac{1}{3} \left(y^2 + 2\right)^{3/2}, 1 \leq y \leq 3 \]

5 pts 3) Find the solution of the differential equation \( (x^2 + 1) \frac{dy}{dx} = xy \) that satisfies the initial condition \( y(1) = 1 \). Solve for \( y \) explicitly for full credit.

5 pts 4) A bacteria culture grows at a rate of \( \frac{dP}{dt} = \lambda P \).

a) Find the solution for \( P(t) \) given the initial condition \( P(0) = P_0 \) (You can guess the solution but show that it satisfies the differential equation)

b) How long does it take for the initial population to double?

5 pts 5) A function \( y(t) \) satisfies the differential equation \( \frac{dy}{dt} = y^4 - 12y^3 + 35y^2 \)

a) Find and plot equilibrium points
b) Determine whether equilibrium points are stable or unstable (or perhaps something else).

Extra Credit (5 pts)

Solve the differential equation.

\[ y' = xe^{-\sin x} - y \cos x \]