Final Exam, Math 30, December 15, 2007

Solve for y explicitly. Indefinite integrals must be in terms of x. Check out useful information in the Appendix.

1) Find the volume of the solid obtained by rotating the given region about the x-axis. **(10 pts)**

Region bounded by: $y = 2x, y = x^2$

2) Find the average value of the function $u(x) = 10x\sin(x^2)$ on the interval $\left[0, \sqrt{\pi}\right]$ **(10 pts)**

Evaluate the following indefinite integrals **(10 pts each)** solutions must be in terms of x.

3) $\int \sin^2(x)\cos^3(x)dx$

4) $\int \frac{1}{x^2\sqrt{x^2-9}}dx$

5) $\int \frac{x+1}{x^2+5x+6}dx$

6) Solve the differential equation that satisfies the given initial condition. (10 pts)

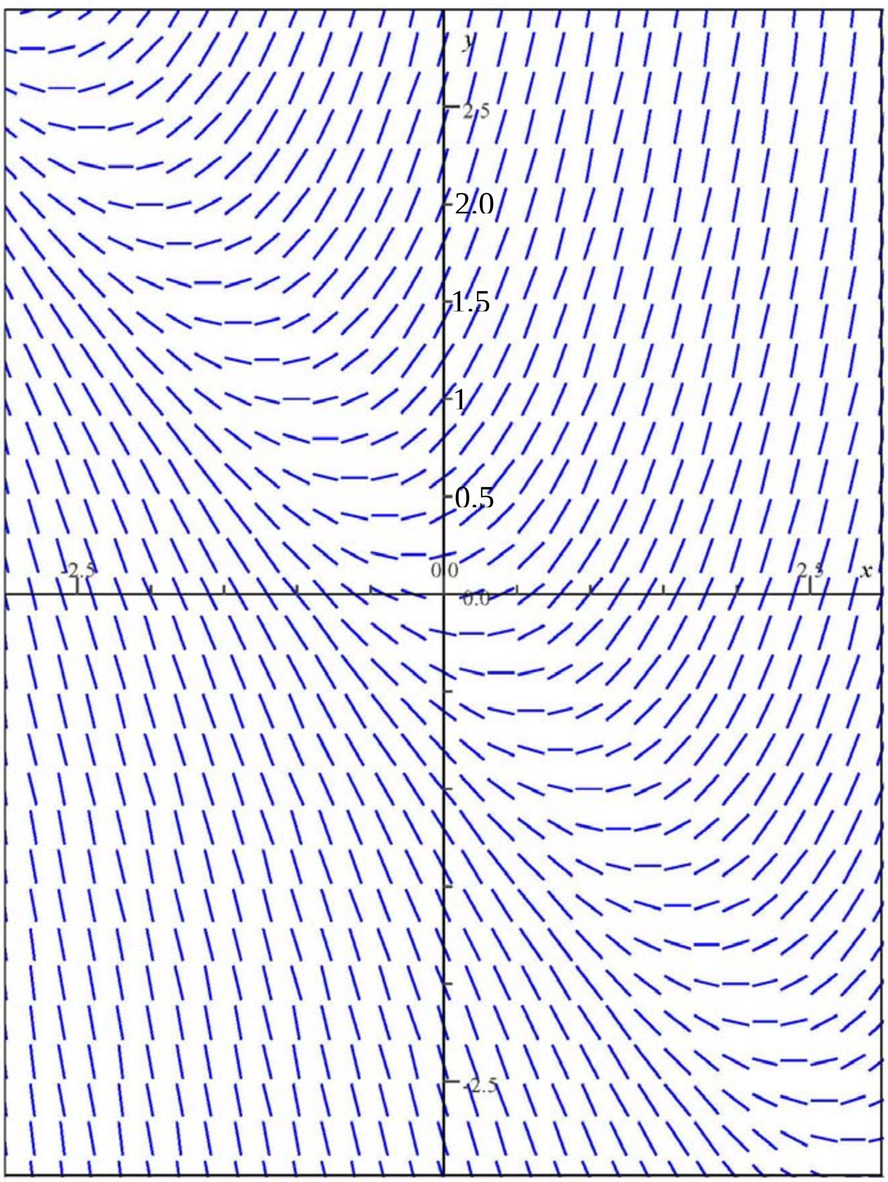
$$\frac{dy}{dt} = y^2 \sin x, \quad y(\frac{\pi}{2}) = 2$$

7) Solve the differential equation: $\frac{dy}{dx} = x + y$ where y(0)=1.0 **(10 pts)**

Sketch a graph of the solutions that satisfy the given initial conditions on the slope field map. **(5 pts)**

i) y(0) = 1.0
ii) y(0) = -1.0

Slope field for y’=x+y



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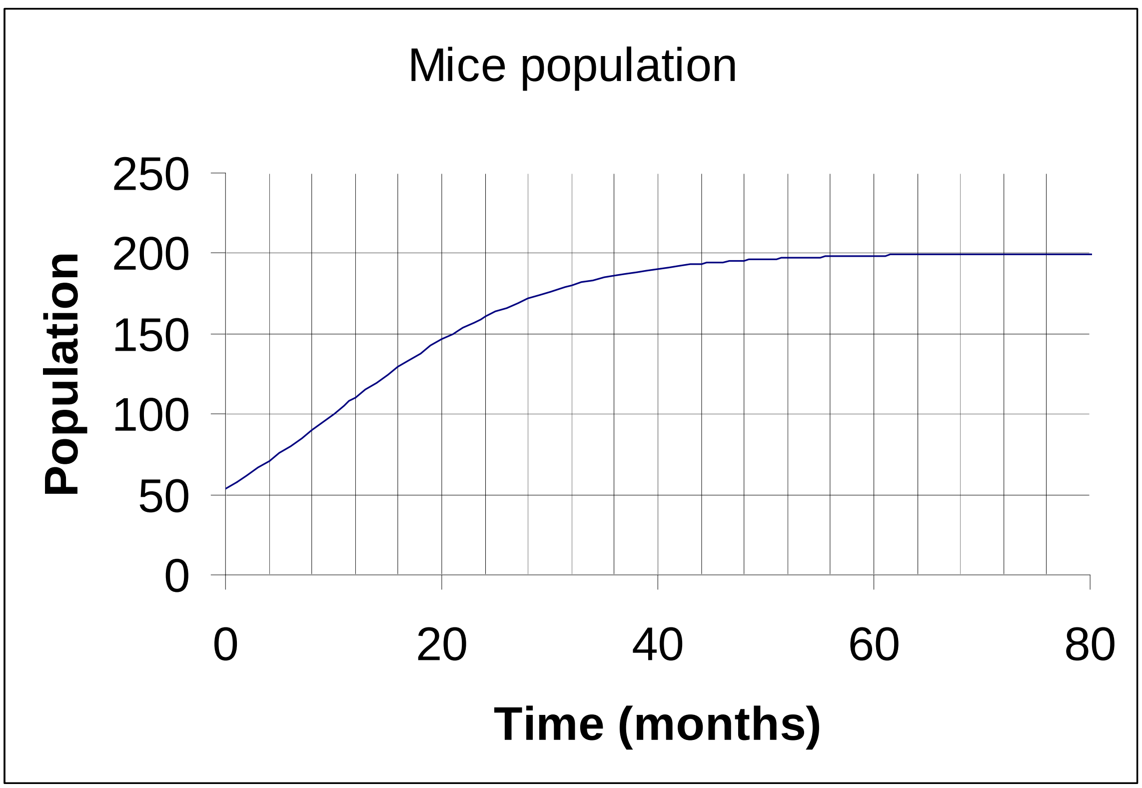
8) A function y(t) satisfies the differential equation:

$$\frac{dy}{dt} = y^4 - 13y^3 + 40y^2$$

a) Find and plot equilibrium points **(10 pts)**
b) Determine whether equilibrium points are stable, unstable (or perhaps something else). **(5 pts)**

9) A graph of mice population is shown below. If the population is described by the Logistics Equation (see the Appendix):

a) Determine constants A and K from the graph **(5 pts)**
b) How long does it take for the initial population to double? **(5 pts)**
c) Estimate the growth rate k constant. *(Hint: What is 1 + Aexp(-kt) when the initial population doubles?)* **(10 pts)**



10) The population of aphids on a rose plant increases at a rate proportional to the number present.

a) Write a differential equation for population of aphids at time t in days. **(10 pts)**
b) Find the solution to the differential equation where at t=0 there were 1000 aphids and population doubles every 10 days. **(10 pts)**

11) Newton's Law of cooling states that the rate of cooling is proportional to the temperature difference between the object and its surrounding. **(10 pts)**

The differential equation for temperature, T, of a coffee cup as a function of time (t) where T_A is the ambient temperature of air, $T_o > T_A$ is the initial temperature of the coffee and k is the proportionality constant is: **(5 pts)**

A) $T' = -kT(1 - T/T_A)$
B) $T' = -k(T - T_A)$
C) $T' = k(T - T_A)$
D) $T' = kT/T_A$
E) $T' = -kT/T_A$

The solution to the differential equation is: **(5 pts)**

A) $T(t) = T_A/(1+A\exp(-kt))$, where $A = (T_A - T_o)/T_o$
B) $T(t) = T_A + T_o\exp(-kt)$
C) $T(t) = T_A\exp(-kt) - T_A + T_o$
D) $T(t) = T_A + (T_o - T_A)\exp(-kt)$
E) $T(t) = (T_o/T_A)\exp(-kt)$

Extra Credit (10 pts)

12) For the following predator pray system determine which of the variables, x or y, represent the prey population and which represent the predator population (Explain). Find equilibrium solutions for preditor and prey.

$$\frac{dx}{dt} = -.05x + .0001xy$$
$$\frac{dy}{dt} = 0.1y - .005xy$$

Appendix:

Identities:

$$\sec^2(x) = \tan^2(x) + 1$$
$$\cos^2(x) + \sin^2(x) = 1$$
$$\sin^2(x) = \frac{1}{2}\left(1 - \cos(2x)\right)$$
$$\frac{d\tan(\theta)}{d\theta} = \sec^2(\theta)$$
$$\frac{d\sec(\theta)}{d\theta} = \tan(\theta)\sec(\theta)$$

Logistics Equation:

$$\frac{dP}{dt} = kP(K - P)$$
$$P(t) = \frac{K}{1 + A\exp(-kt)}$$
$$A = \frac{(K - P_0)}{P_0}$$

ln(AB)=ln(A) + ln(B)
ln(2)=0.693
ln(3)= 1.10
ln(4)=1.39
ln(5)=1.61