1) Find the volume of a solid when the region bounded by $y = \sin(x)$ and $y=0$ is revolved around the y-axis (10 pts)

2) An enclosure for rare reptiles must maintain an average temperature of 30C. The temperature profile during the day is given by $T(t) = T_m \cos\left(\frac{\pi}{48}t\right)$ where $t$ is in hours. Find $T_m$ so that the average temperature during 24 hours is 30C. (10 pts)

Evaluate the following integrals: Answer must be in terms of $x$

3) $\int \frac{\sqrt{x^2 - 1}}{x} \, dx$ (10 pts)

4) $\int \frac{3x + 5}{x^2 + x - 6} \, dx$ (10 pts)

5) A tank with 2000 gal of water is being drained at the rate given in the diagram. How much water is left in the tank after 50 min? (12 pts)
(1) \[ V = \pi \int_0^\pi 2y \, dy \]
\[ V = 2\pi \int_0^\pi x \sin x \, dx \]
\[ u = x, \quad dv = 2\sin x \, dx \]
\[ du = dx, \quad v = -2\cos x \]
\[ V = 2\pi \left( -x\cos x \bigg|_0^\pi + \int_0^\pi \cos x \, dx \right) \]
\[ = 2\pi \left( -\pi \cos \pi + 2\sin x \right) \bigg|_0^\pi = 2\pi \left( \pi \right) = 2\pi^2 \]

(2) \[ T_{av} = \frac{1}{24} \int_0^{2\pi} T_m \cos \left( \frac{\pi t}{18} \right) \, dt = 30 \]
\[ = \frac{T_m}{24} \left[ \frac{18}{\pi} \sin \left( \frac{\pi t}{18} \right) \right]_0^{2\pi} = \frac{2T_m}{\pi} = 30 \]
\[ T_m = 15\pi \]
(3) \[ \int \frac{\sqrt{x^2-1}}{x} \, dx \quad x = \sec \theta \]
\[ \frac{d\theta}{\sec \theta \tan \theta} = \frac{d\theta}{\sec \theta} \]
\[ = \int \frac{\sec \theta \tan \theta}{\sec \theta} \, d\theta = \int \tan^2 \theta \, d\theta \]
\[ = \int (\sec^2 \theta - 1) \, d\theta = \tan \theta - \theta \]
\[ = \sqrt{x^2-1} - \sec \theta \left( \frac{1}{x} \right) + C \]

(4) \[ \int \frac{3x+5}{x^2+x-6} \, dx \]
\[ \frac{3x+5}{x^2+x-6} = \frac{3x+5}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2} \]

Common denominator: \[ 3x+5 = A(x-2) + B(x+3) \]
\[ 3x+5 = (A+B)x + (3B-2A) \]
\[ A+B = 3 \quad \text{and} \quad -3A - 3B = 5 \]
\[ 3B - 2A = 5 \]
\[ -5A = -4 \]
\[ A = \frac{4}{5} \]

\[ B = 3 - A = 3 - \frac{4}{5} = \frac{11}{5} \]

\[ \int \frac{(3x+5)}{x^2+x-6} \, dx = \frac{4}{5} \ln |x+3| + \frac{11}{5} \ln |x-2| + C \]
Amount of water drained from tank
\[ \int_0^{30} \gamma(t) \, dt \quad \gamma(t) = \begin{cases} 30 & 0 \leq t < 30 \\ -(t-60) & t \geq 30 \end{cases} \]

\[ \int_0^{50} \gamma(t) \, dt = \int_0^{30} 30 \, dt + \int_{30}^{50} (60-t) \, dt \]

\[ = 30t \bigg|_0^{30} + \left( 60t - \frac{t^2}{2} \right) \bigg|_{30}^{50} \]

\[ = 900 + \left( 3000 - \frac{2500}{2} - 1800 + \frac{900}{2} \right) \]

\[ = 1300 \]

Amount of water left \[ 2000 - 1300 = 700 \text{ gal} \]