Solution pg 2 and 3

Midterm 3, Math 30, Fall 2008, 12/08/08

Instructions: Write your name and section number. Draw grading table on the cover. Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. Solutions must be simplified as much as possible, no full credit for partially completed problems. **Blue books with torn or missing pages will not be accepted !**

1. (10 pts) Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = 3$ and $y(1) = 2$
2. (10 pts) The population of aphids on a rose plant increases at the rate proportional to the number present. After 1 day the population increased from 100 to 101. How long will it take for population to get to 1000?
3. Write down but don't solve differential equations for following problems. Let the proportionality constant be $k > 0$.
   a. (5 pts) A glucose solution is administered intravenously into the bloodstream at a constant rate r. The glucose is removed from the bloodstream at a rate proportional to the concentration at that time. Write down the differential equation for concentration over time dC/dt where C is the concentration at time t.
   b. (5 pts) A hot object cools off at the rate proportional to the temperature difference between the object and surrounding air. Let T(t) be the temperature of the object and T_A be the ambient temperature of surrounding air ($T > T_A$). Write down the differential equation for dT/dt
4. (20 pts) The golden toad of Costa Rica became extinct in 1989; the first documented casualty of global warming. Let's model the toad population in the pond by differential equation $P' = .05P - 20$ where starting population is 300 toads and time t is measured in years. Solve the equation and calculate how long it took for the toad to become extinct.

Extra Credit: In problem 2 assume logistic growth with carrying capacity of 2000. Write down the logistic equation. (5 pts)

MATH 30 MIDTERM 3 SOLUTION

① $\frac{dy}{dx} + \frac{y}{x} = 3$ $\quad p(x) = \frac{1}{x} \quad q(x) = 3$

$y(1) = 2$ $\quad I(x) = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$$y = \frac{\int I q(x)\,dx}{I} = \frac{\int 3x\,dx}{x} = \frac{3x^2}{2x} + \frac{c}{x}$$

$$y = \frac{3x}{2} + \frac{c}{x} \qquad y(1) = 2 = \frac{3}{2} + c$$

$$2 - \frac{3}{2} = c$$

$$c = \frac{1}{2}$$

$$\boxed{y = \frac{3x}{2} + \frac{1}{2x}}$$

② $\frac{dP}{dt} = kP$ $\quad P(t) = P_0 e^{kt}$

$101 = 100 e^{k}$

$1.01 = e^{k}$

$\ln(1.01) = k$

$1000 = 100 e^{kT}$

$10 = e^{kT}$

$\ln 10 = kT$

$T = \frac{\ln 10}{\ln(1.01)} = 1005$ days

③ a) $\frac{dc}{dt} = \frac{r}{V} - kc$ where V volume of blood (assuming constant) $\frac{dc}{dt} = r - kc$ is also accepted

b) $\frac{dT}{dt} = -k(T - T_A)$ since $T - T_A > 0$ rate is negative indicating cooling

(4) $\frac{dP}{dt} = .05P - 20$

$P' - .05P = -20 \qquad I(t) = e^{\int -.05\,dt} = e^{-.05t}$

$P(t) = \dfrac{-\int 20e^{-.05t}\,dt}{e^{-.05t}} = \dfrac{400e^{-.05t} + C}{e^{-.05t}}$

$P(t) = 400 + Ce^{.05t}$

$P(0) = 300 = 400 + C \qquad C = -100$

$P(t) = 400 - 100e^{.05t}$

Extinction when $P(T) = 0$

$0 = 400 - 100e^{.05T}$

$4 = e^{.05T} \qquad T = \dfrac{\ln 4}{.05} = 27.7\text{yrs}$

Extra credit

$k = \ln\left(\frac{101}{100}\right)$

$\frac{dP}{dt} = kP\left(1 - \frac{P}{2000}\right)$