Final Exam, Math 30, Fall 2008, 12/13/08

Instructions: Write your name and section number. Draw grading table on the cover. Read each problem carefully and follow all of its instructions. For each of the problems below, write a clear and concise solution in your blue book. Solutions must be simplified as much as possible, no full credit for partially completed problems. **Blue books with torn or missing pages will not be accepted !**

1. (10 pts) Find the average of $f(x) = \sin^2(x)\cos^3(x)$ on interval $[-\pi, \pi]$
2. (10 pts) Derive the given formula where n and a are constants.
   $$\int x^n \cos(ax)dx = \frac{1}{a}x^n \sin(ax) - \frac{n}{a}\int x^{n-1}\sin(ax)dx$$
3. (10 pts) Sketch the region bounded by $y=x^3$, x=0 and y = 1. Find the volume of revolution when the region is revolved about the x-axis.
4. (10 pts) Find the length of the curve described by the function
   $y = \frac{x^2}{8} - \ln(x)$ from x =1 to x = 4.
5. (10 pts) Solve the differential equation y' + cos(x)y=cos(x), where y(0)=2
6. (10 pts) A manager of a fast food restaurant advertises that any customer waiting for more than X minutes will get a free meal. The mean waiting time is 5 min. What should she set X to so that no more than 1% of customers get a free meal?
7. Virions (virus particles) in an infected patient increase at the rate proportional to the virion number. $\frac{dV}{dt} = kV$. Suppose that at t=0 (Measured in days) the patient begins to take antivirus medication that eliminates virions at the rate r. The elimination rate is related to the daily medicine dose by equation r = aD. Let k =.1/day, a = 200 /(day mg) , V(0)=100000.
   a. (5 pts) Solve the equation $\frac{dV}{dt} = kV - r$
   b. (5 pts) What minimum dose does the patient need to take so that virion number decreases over time? (Hint: Write the answer as inequality D > ?)
8. A climate model for average annual global temperature(in Fahrenheit) is given by: $\frac{dT}{dt} = T^2(T-68)(T-86)(104-T)$
   a. (8 pts) Find and identify by type all equilibrium points.
   b. (7 pts) Suppose that the current average annual global temperature is 77 F. Suppose that current CO_2 emissions are projected to increase this temperature by 11F. Is there a major risk? Using equilibrium points, explain what might happen.
9. Suppose population of wolves and rabbits are modeled with the following Lotka-Voltera equations.

$$\frac{dx}{dt} = -.02x + 2X10^{-5}xy$$
$$\frac{dy}{dt} = .1y - .001yx$$

a. (5 pts) Determine which variable x or y represents rabbits and which represents wolves. Explain
b. (5 pts) Find equilibrium solutions.
c. (5 pts) Sketch the phase trajectory corresponding to the initial population of 100 wolves and 500 rabbits. Indicate the direction.

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Extra Credit:

(5 pts) If the patient in Problem 7 wants to eliminate all virions in 100 days, how big should his daily dose be?