MIDTERM 3 SOLUTION 4/28/08

(1) Average waiting time $\mu = 2$ min
probability that a customer will wait for more than 5 min

$$P(t \geq 5) = \frac{1}{2}\int_5^\infty e^{-t/2}\,dt = \lim_{h\to\infty} \frac{1}{2}\int_5^h e^{-t/2}\,dt$$

$$= \lim_{h\to\infty} -1\left[e^{-h/2} - e^{-5/2}\right].$$

$$= e^{-5/2} \qquad \lim_{h\to\infty} e^{-h/2} = 0$$

To convert to percentage multiply fraction by 100

$100\,e^{-5/2}$ % of customers will get a free drink 8.2% !

(2) $$\frac{dy}{dx} = y^2 x$$

separate variable and integrate both sides

$$\int \frac{dy}{y^2} = \int x\,dx$$

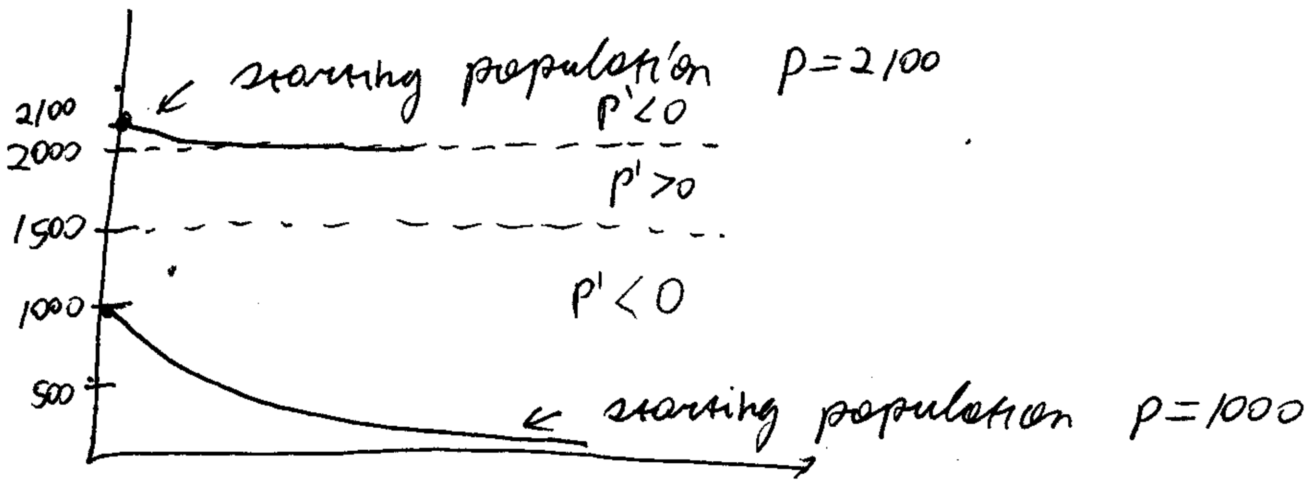
$$-y^{-1} = \frac{x^2}{2} + c$$, y passes through $(1,-1)$

$$+1 = \frac{1}{2} + c \qquad c = \frac{1}{2}$$

$$\boxed{y = \frac{1}{-\left(\frac{x^2}{2} + \frac{1}{2}\right)}}$$

③ $\frac{dp}{dt} = p^2(p-1500)(2000-p)$

Equilibrium points occur at $\frac{dp}{dt} = 0$
$p = 0, 1500, 2000$



a) If the starting population is 1000 over time it will crash to zero
b) If the starting population is 2100 over time it will stabalize at 2000

④ a) $\frac{dI}{dt} = kI(1-I)$ $\quad k>0$
I = number of infected cells

b) $\frac{dN}{dt} = -kN$ $\quad k>0$
N = number of radioactive atoms.

⑤ $\frac{dP}{dt} = \lambda P$ is exponential growth function

$P(t) = P_0 e^{\lambda t}$ $\quad$ when P_0 doubles
$2\cancel{P_0} = \cancel{P_0} e^{\lambda t_2}$ $\quad$ $\ln 2 = \lambda t_2$

$$\boxed{t_2 = \frac{\ln 2}{\lambda} = 69 \text{ hours}}$$