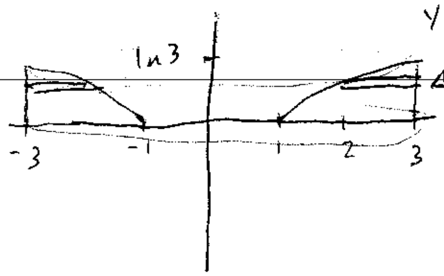
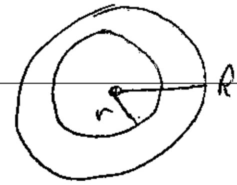
$x = e^y$
$y = \ln x$

1





$R = 3$
$r = e^y$

$A = \pi(3^2 - e^{2y})$

$\Delta V = \pi(9 - e^{2y})\,\Delta y$

$V = \int_0^{\ln 3} \pi(9 - e^{2y})\,dy$

$V = \pi\left(9y - \frac{e^{2y}}{2}\right)\Big|_0^{\ln 3}$

$V = \pi\left(9\ln 3 - \frac{9}{2} + \frac{1}{2}\right)$

$V = \pi(9\ln 3 - 4)$

$V = (9\ln 3 - 4)\pi$

2

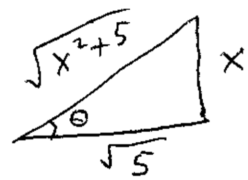
a) $\int \frac{\sin(\ln x)}{x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$

$\int \sin(u)\,du$

$-\cos(u) + c$

$-\cos(\ln x) + c$

b) $\int \frac{1}{x^2\sqrt{x^2+5}}\,dx$ $\quad x = \sqrt{5}\tan\theta$, $dx = \sqrt{5}\sec^2\theta\,d\theta$



$\int \frac{1}{5\tan^2\theta\sqrt{5\tan^2\theta + 5}}\sqrt{5}\sec^2\theta\,d\theta$

$\frac{1}{5}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta \rightarrow \frac{1}{5}\int \frac{\cos^2\theta}{\sin^2\theta\cos\theta}\,d\theta$ $\quad u = \sin\theta$, $du = \cos\theta\,d\theta$

$\frac{1}{5}\int \frac{1}{u^2}\,du$

$-\frac{1}{5}\frac{1}{u} \rightarrow -\frac{1}{5}\frac{1}{\sin\theta} \rightarrow -\frac{\sqrt{5+x^2}}{5x}$

$-\frac{\sqrt{x^2+5}}{5x} + C$

3

$\int_{-\infty}^{\infty} p(x)\,dx = 1 \rightarrow \int_0^5 p(x)\,dx = 1$

$c\int_0^5 5x - x^2\,dx = 1$

$c\left(\frac{5x^2}{2} - \frac{x^3}{3}\right)\Big|_0^5 = 1$

$c\left(\frac{125}{2} - \frac{125}{3}\right) = 1$

$c\,\frac{125}{6} = 1$

$c = \frac{6}{125}$

4 $\int \frac{1}{y}\,dy = \int \frac{1}{x^2+5x+6}\,dx$ $\qquad \frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$

$\ln y = \int \frac{1}{x+2} - \frac{1}{x+3}\,dx$ $\qquad 1 = A(x+3) + B(x+2)$

$\ln y = \ln(x+2) - \ln(x+3) + C$ $\qquad x=-3 \rightarrow 1 = -B \quad B=-1$

$y(0)=2 \rightarrow \ln 2 = \ln(2) - \ln(3) + C$ $\qquad x=-2 \rightarrow 1 = A$

$C = \ln 3$

$\ln y = \ln\left(\frac{3(x+2)}{x+3}\right) + \cancel{C}$

$y = \frac{3(x+2)}{x+3}$

---

5 $\frac{dm}{dt} = -\lambda m$ $\qquad m = m_0 e^{-\lambda t}$

$\int \frac{1}{m}\,dm = \int -\lambda\,dt$ $\qquad .66\,m_0 = m_0 e^{-\lambda(3)}$

$\ln m = -\lambda t + c$ $\qquad \frac{\ln .66}{3} = \lambda$

$m = e^{-\lambda t} e^{c}$

$t_{1/2} = \frac{3 \ln 1/2}{\ln .66} = 5.004$

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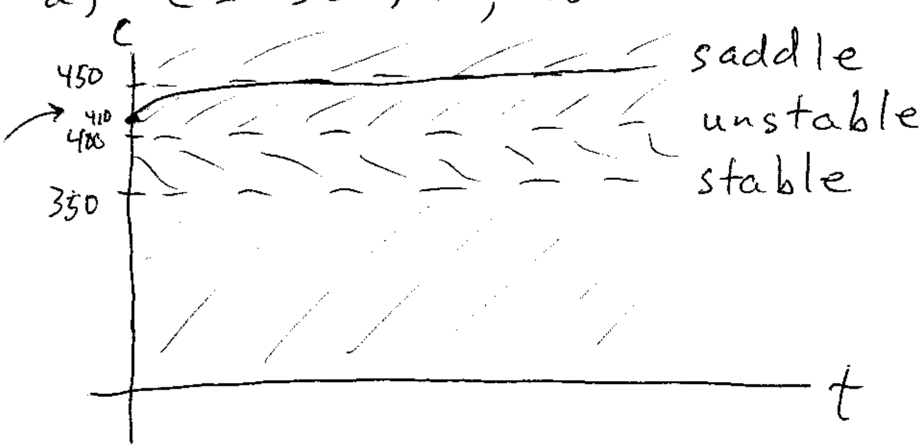
6 $\frac{dC}{dt} = k\,(C-450)^2 (C-400)(C-350)$

a) $C = 350, 400, 450$

b)



c) unstable - a small perturbation would send it up and away

d) reduce by more than 10ppm so $C < 400$

$\boxed{7}$ $\frac{dP}{dt} = \lambda P + m \qquad P(0) = P_0$

a) $\int \frac{1}{\lambda P + m}\, dP = \int dt$

$\ln(\lambda P + m) = t + c$

$P(0) = P_0 \to \ln(\lambda P_0 + m) = c$

$\ln(\lambda P + m) = t + \ln(\lambda P_0 + m)$

$\lambda P + m = (\lambda P_0 + m) e^t$

$P = \frac{(\lambda P_0 + m) e^t - m}{\lambda}$

b) $\lambda > \frac{-m}{P} \quad \Leftarrow \lambda P + m > 0$

c) $\lambda P + m < 0 \qquad \lambda < \frac{-m}{P}$

d) $\lambda = \frac{-m}{P}$

$\boxed{8}$ a) $\frac{dH}{dt} = -kH(1-H)$

b) $\frac{dT}{dt} = -k(T - T_a)$

$\boxed{9}$ x is predator
y is prey

$\frac{dx}{dt} = -.02x + .00002xy = 0 \to -2000x + 2xy = 0$

$x(-2000 + 2y) = 0$

$x = 0, \quad y = 1000$

$\frac{dy}{dt} = .1y - .001xy = 0 \to 100y - xy = 0$

$y(100 - x) = 0$

$y = 0 \qquad x = 100$

$x = 0, \ y = 0$

$x = 100, \ y = 1000$