Duration: 3 hours
Instructions: A cheat sheet on one side of a 8.5x11″ page is allowed and must be turned in with the exam. A calculator is allowed when adding, subtracting, multiplying, dividing, and calculating the mean and standard deviation. For everything else, show your work. Partial credit will be awarded for correct work. The total number of points is 100.

1. (10 points)A survey of a group's viewing habits over the last year revealed the following information:

   (i) 29% watched baseball
   (ii) 19% watched soccer
   (iii) 12% watched baseball and soccer

   (a) Given that a group member watched baseball, what is the probability that (s)he also watched soccer?
   (b) What percentage of the group watched neither baseball nor soccer?
   (c) Are the following events independent and why: (I) a given group member watched baseball and (II) a given group member watched soccer?

2. (12 points) Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} ax^3 & 0 < x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the exact value of a.
   (b) Compute $\mathrm{E}[X]$.
   (c) Compute $\mathrm{Var}(X)$.
   (d) Compute the cumulative distribution function $F(x)$.

3. (8 points) The time X, in hours, it takes to repair a personal computer is a random variable uniformly distributed over $[0, 2]$. The cost, in dollars, of the repair depends on the time it takes and is equal to $40 + 30X$. Compute the expectation and variance of the cost to repair a personal computer.

4. (8 points) The number of years a radio functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Jones buys a functioning 4–year old radio, what is the probability that it is still be working after an additional 10 years?

5. (10 points) A small pond contains 1000 fish, 100 of which are tagged. Suppose that 20 fish are caught. Write down an expression to compute the exact probability that there are at least 2 tagged fish among the caught.

6. (10 points) Each computer chip made in a certain plant will, independently, be defective with probability 0.25. A sample of 1000 chips is tested and we are interested in the probability that 200 or fewer chips are found to be defective.

   (a) Write down an expression to compute this probability exactly.
   (b) Approximate this probability using the Central Limit Theorem.

7. (10 points) The IQ score of all students at a large university is normally distributed. The following are scores on IQ tests of a random sample of 9 students.

   130 122 119 142 136 127 120 152 141

   (a) Find the average, median and standard deviation of this sample data.

   (b) Construct a 95% confidence interval estimate of the average IQ score of all students.

   (c) Construct a 95% lower confidence interval estimate.

8. (10 points) In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20. A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of 0.02. Suppose 10 independent measurements yielded the following pH values:

8.18	8.17
8.21	8.20
8.17	8.21
8.22	8.17
8.19	8.18

   What conclusion can be drawn at the $\alpha = 0.1$ level of significance?

9. (10 points) Let $a > 0$ be a parameter and define

$$f(x) = \frac{a}{x^{a+1}}, \quad x \geq 1.$$

   $f(x)$ is a probability density function. It is called a *Pareto distribution with shape parameter* a, and is named for the economist Vilfredo Pareto. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample of Pareto distributions with shape parameter a, find the maximum likelihood estimator of a.

10. (12 points) The number of times that an individual contracts a cold in a given years is a Poisson random variable with parameter $\lambda = 3$. Suppose a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 2$ for 75% of the population. For the other 25% of the population, the drug has no appreciable effect on colds. An individual tries the drug for a year and has 0 colds in that time.

    (a) Assuming that the drug is beneficial for him, what is the probability that he has 0 cold in a year?

    (b) Assuming that the drug is not beneficial for him, what is the probability that he has 0 cold in a year?

    (c) How likely is it that the drug is beneficial for him?

TABLE A1 *Standard Normal Distribution Function:* $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE A3 *Values of $t_{\alpha,n}$*

n	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.474	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576

Other t Probabilities:
$P\{T_8 < 2.541\} = .9825$ $P\{T_8 < 2.7\} = .9864$ $P\{T_{11} < .7635\} = .77$ $P\{T_{11} < .934\} = .81$ $P\{T_{11} < 1.66\} = .94$ $P\{T_{12} < 2.8\} = .984$.