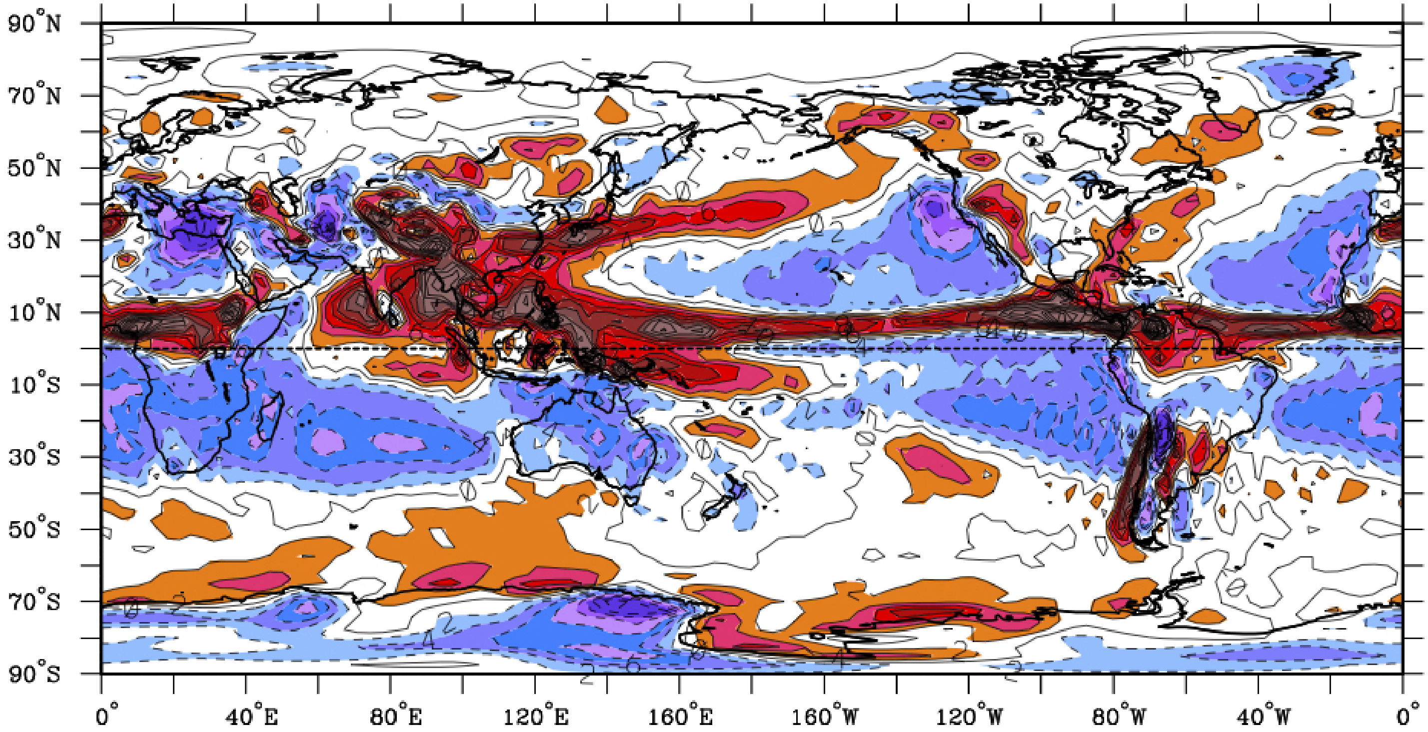
June 40 year mean vertical velocity



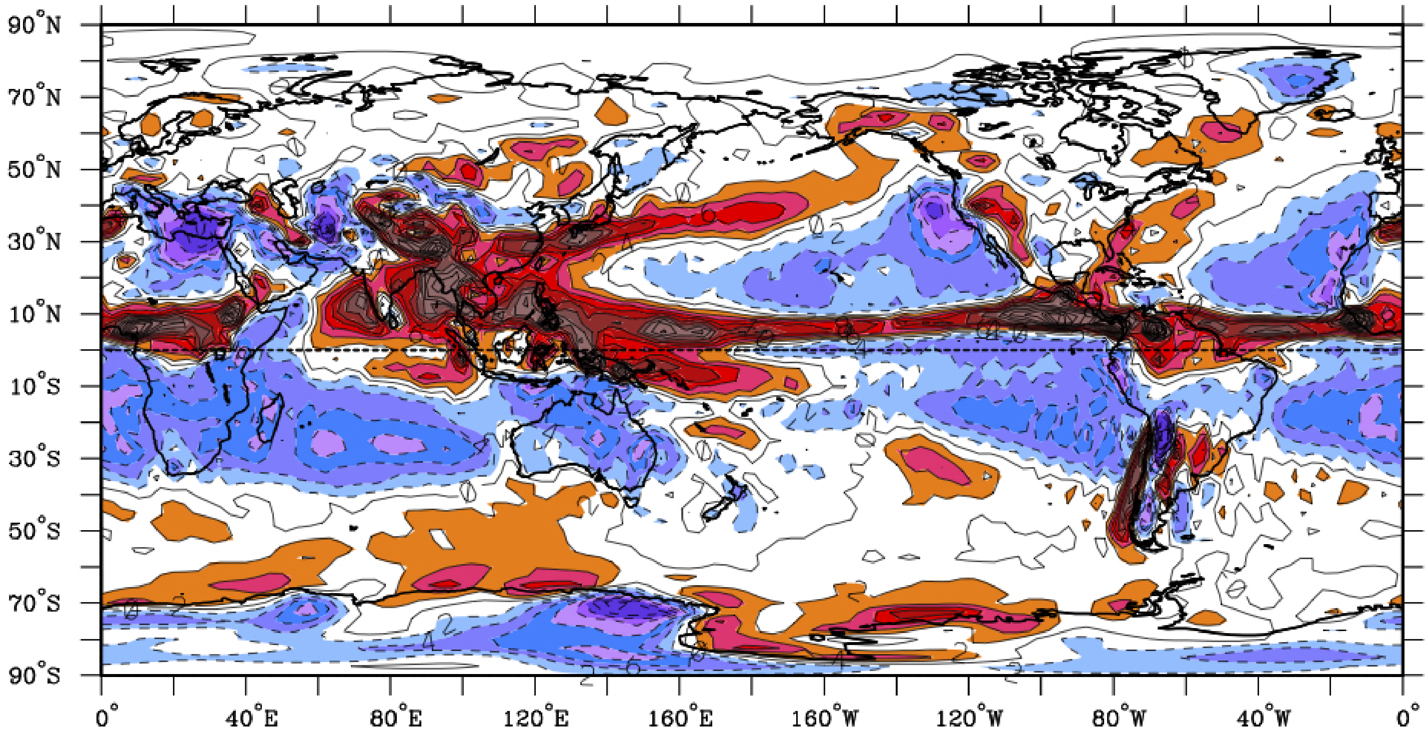
PDEs and Asymptotics for the Tropical Atmosphere

Joseph A. Biello

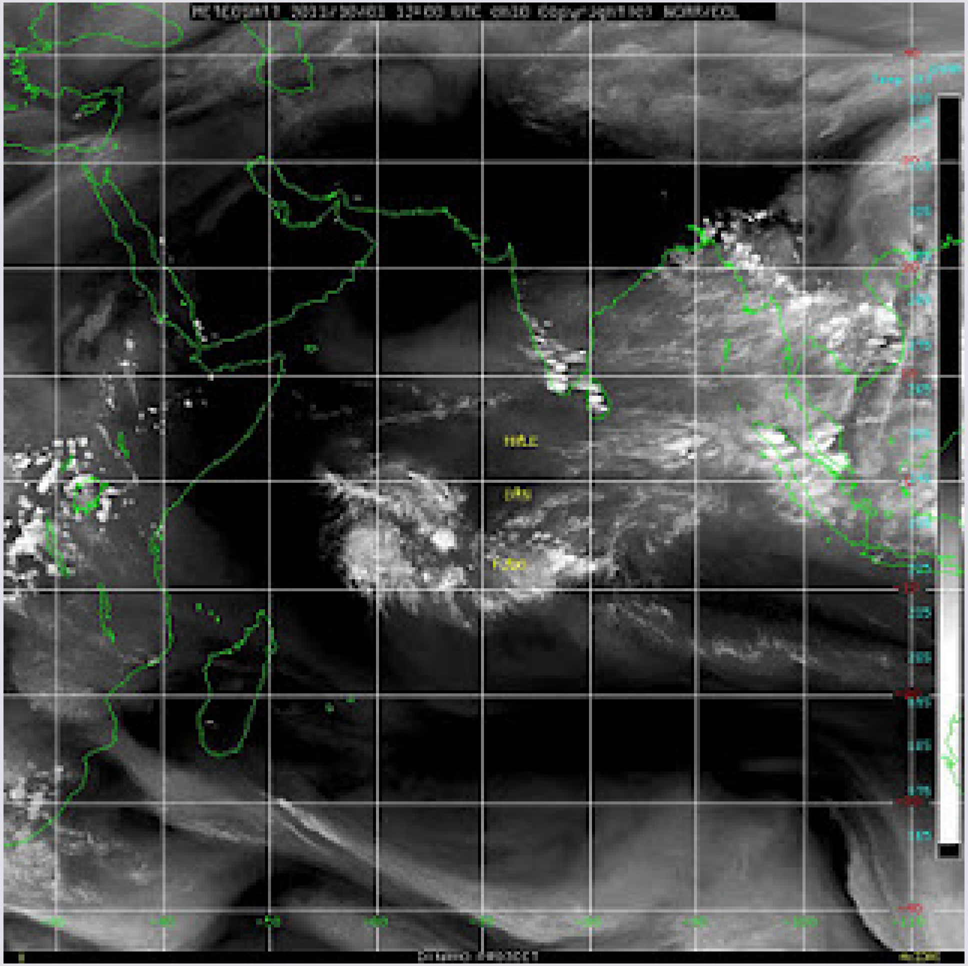
- joint work with Andy Majda (NYU), Francesco Rubini (U. Florence), Paolo Ruggieri (U. Bologna), Xiaoyun Niu (UC Davis)
- Multiple scales wave/mean flow theory of the Madden-Julian oscillation
- Interpreting the results of the ECMWF climate model using the multi-scale theory.

UC DAVIS
UNIVERSITY OF CALIFORNIA

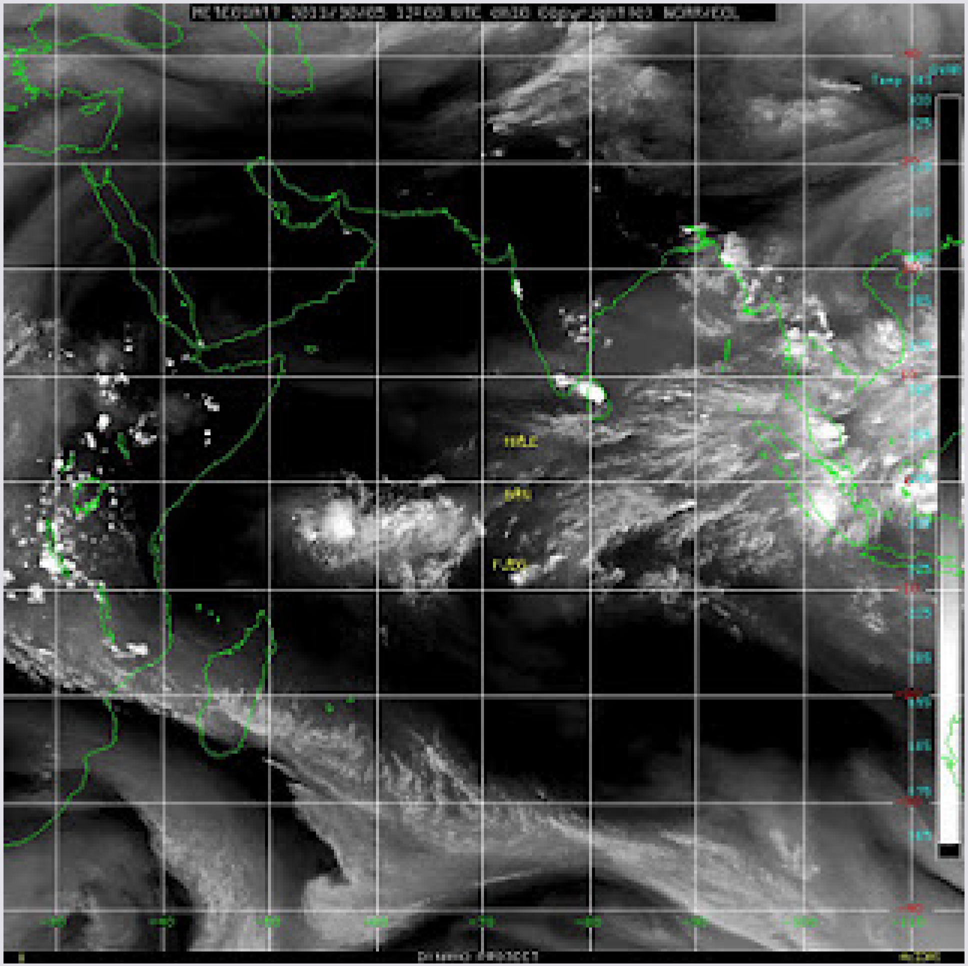
June 40 year mean vertical velocity



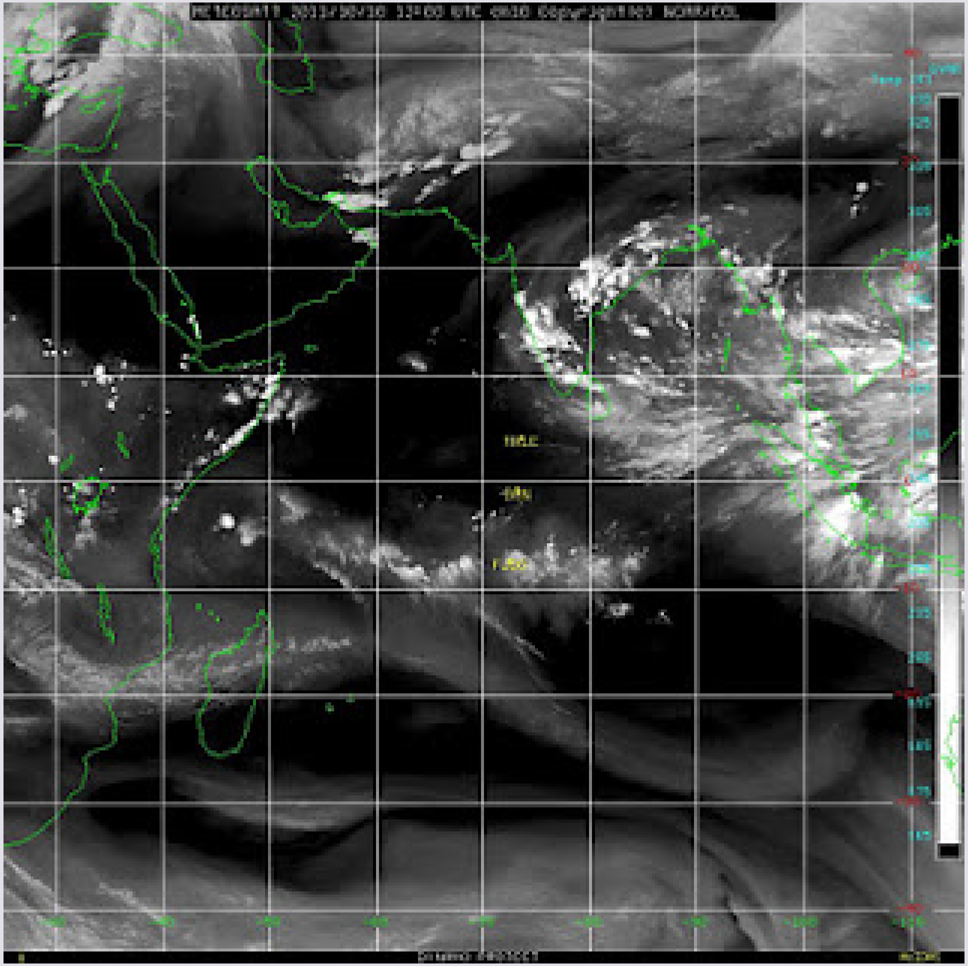
MJO in the Infrared: DYNAMO , Oct 1, 2011



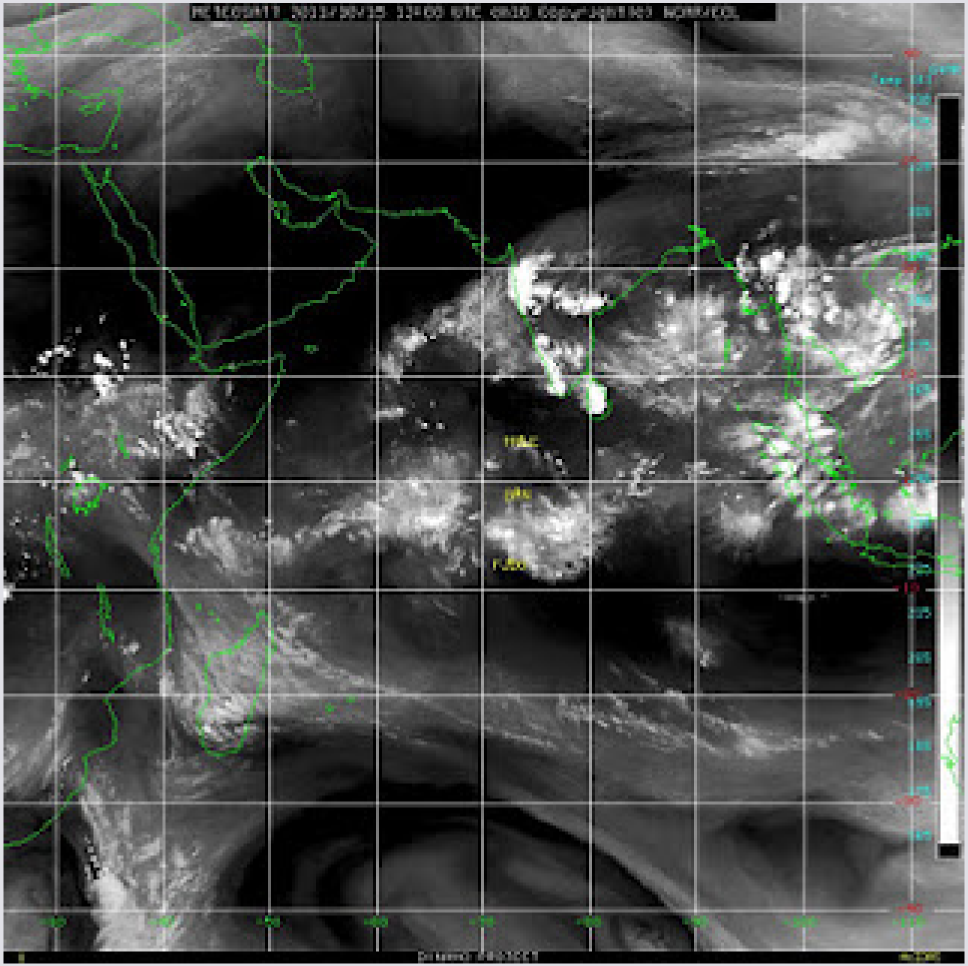
MJO in the Infrared: DYNAMO , Oct 5, 2011



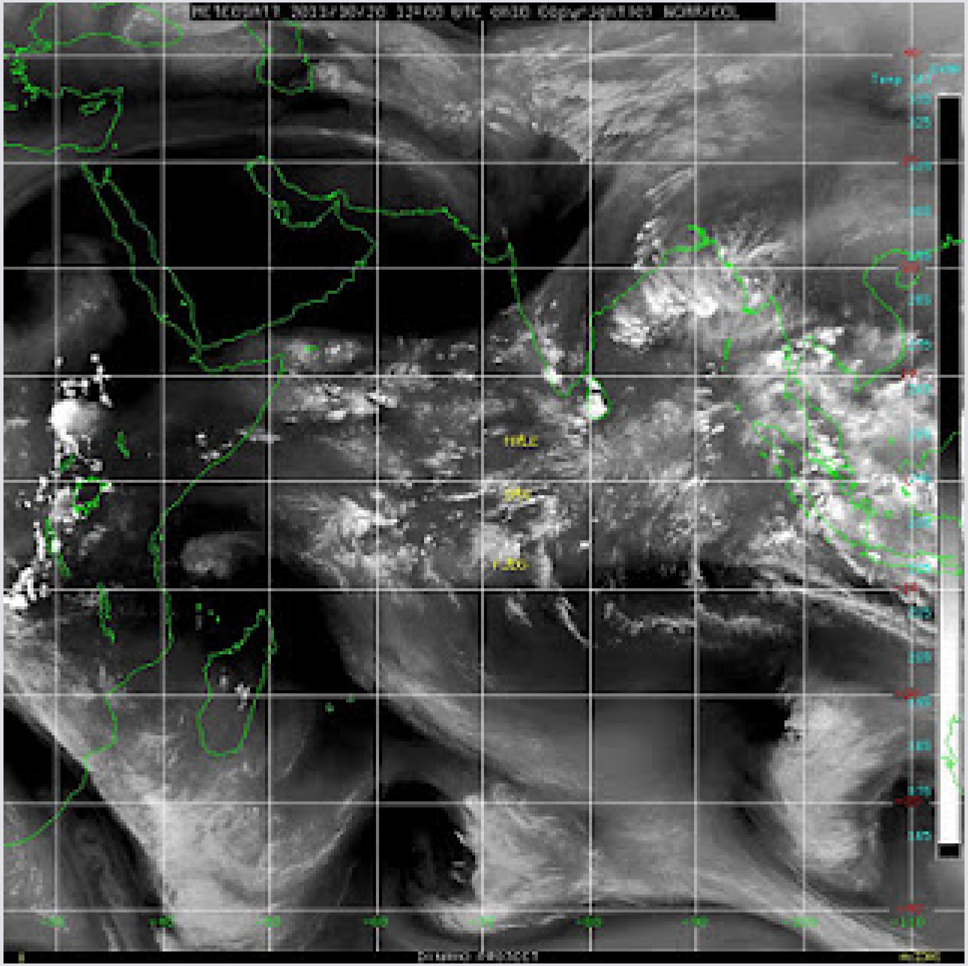
MJO in the Infrared: DYNAMO , Oct 10, 2011



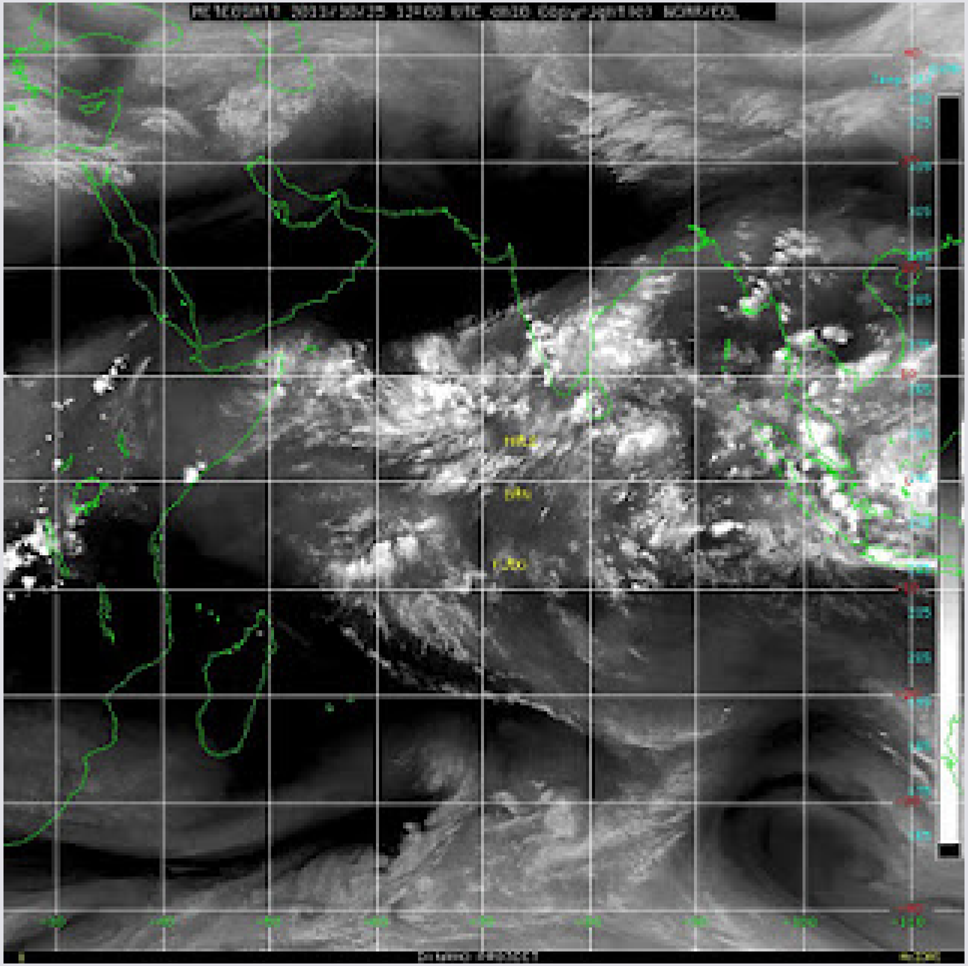
MJO in the Infrared: DYNAMO , Oct 15, 2011



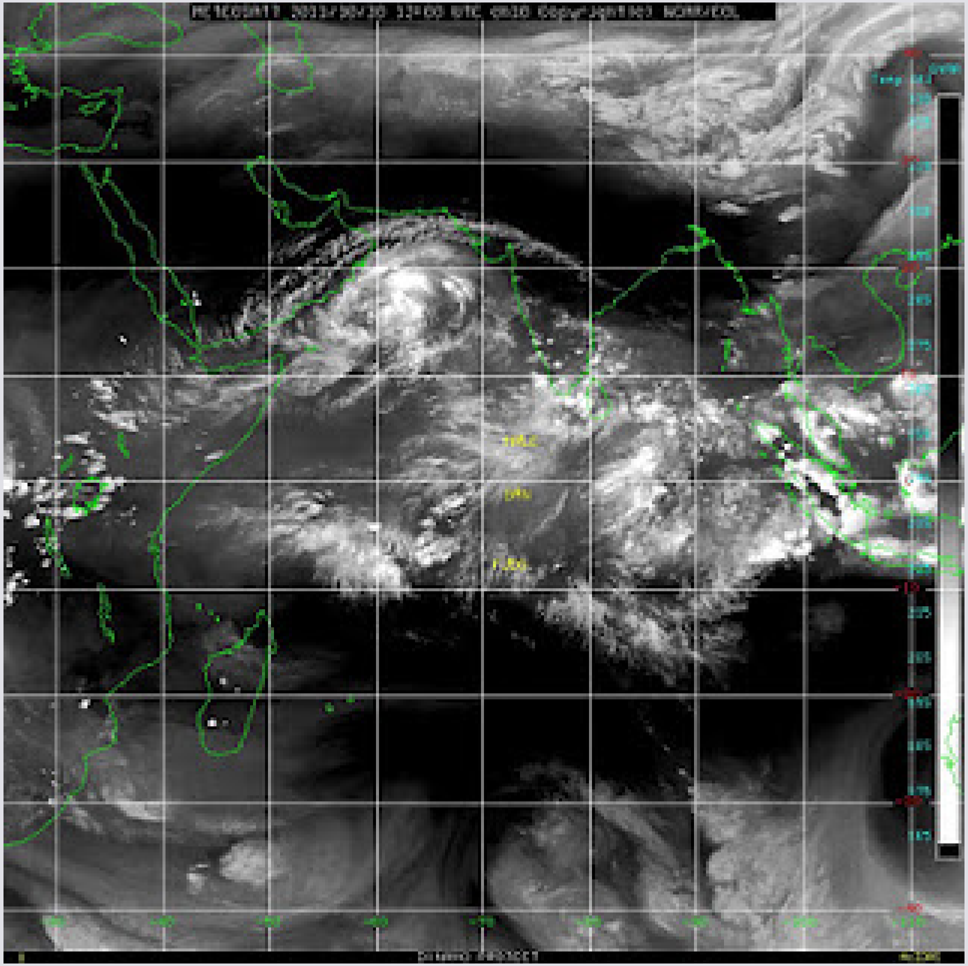
MJO in the Infrared: DYNAMO , Oct 20, 2011



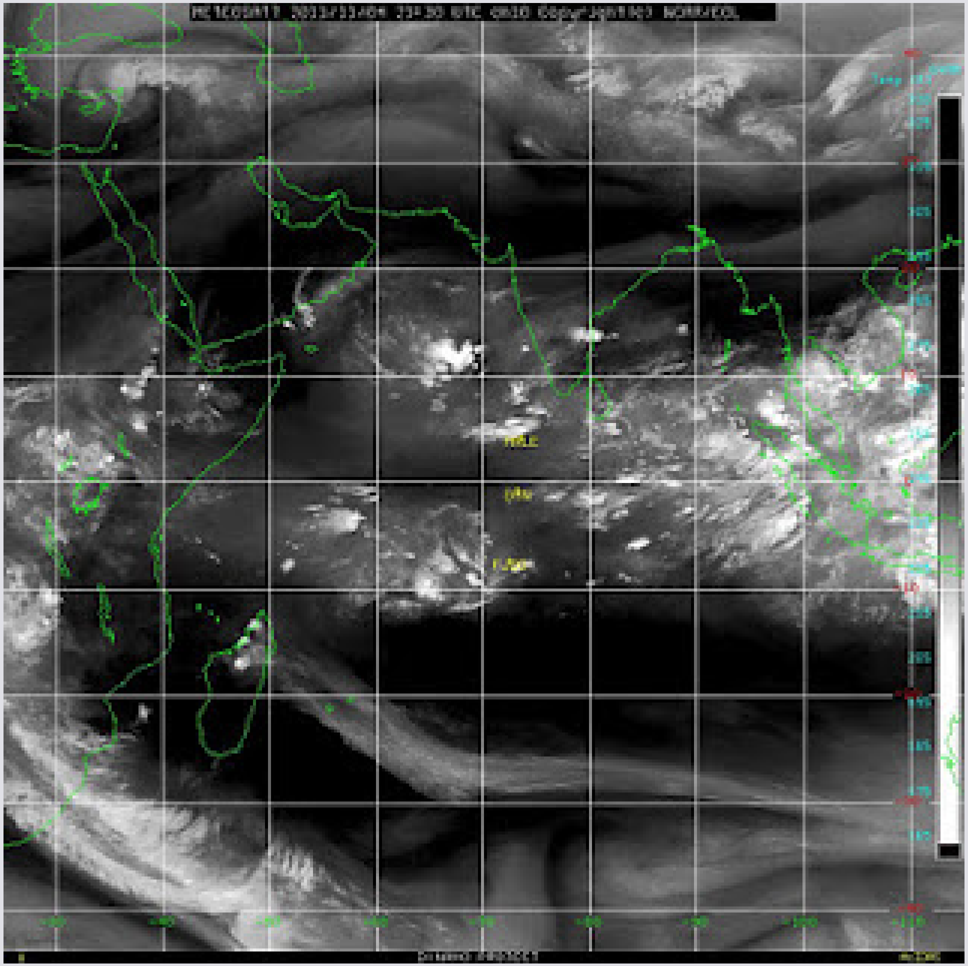
MJO in the Infrared: DYNAMO , Oct 25, 2011



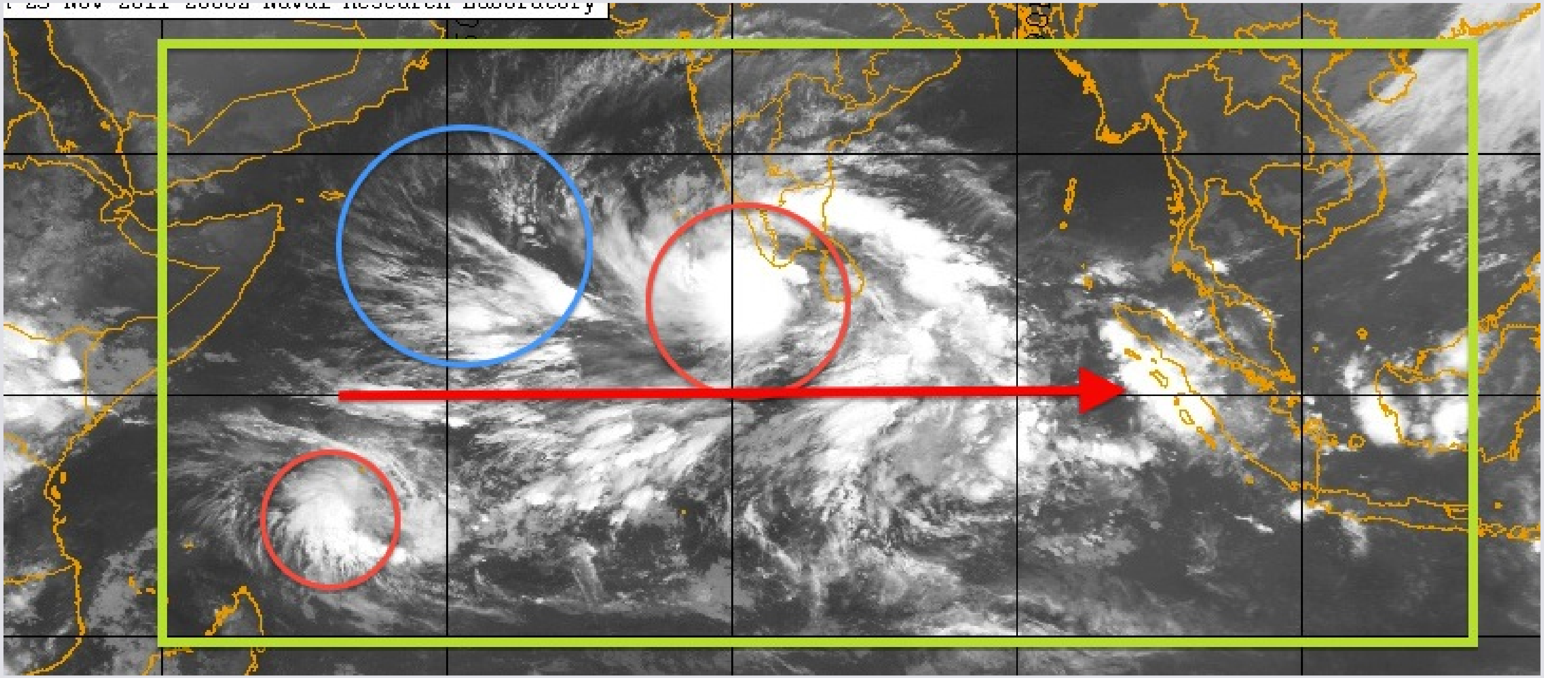
MJO in the Infrared: DYNAMO , Oct 30, 2011



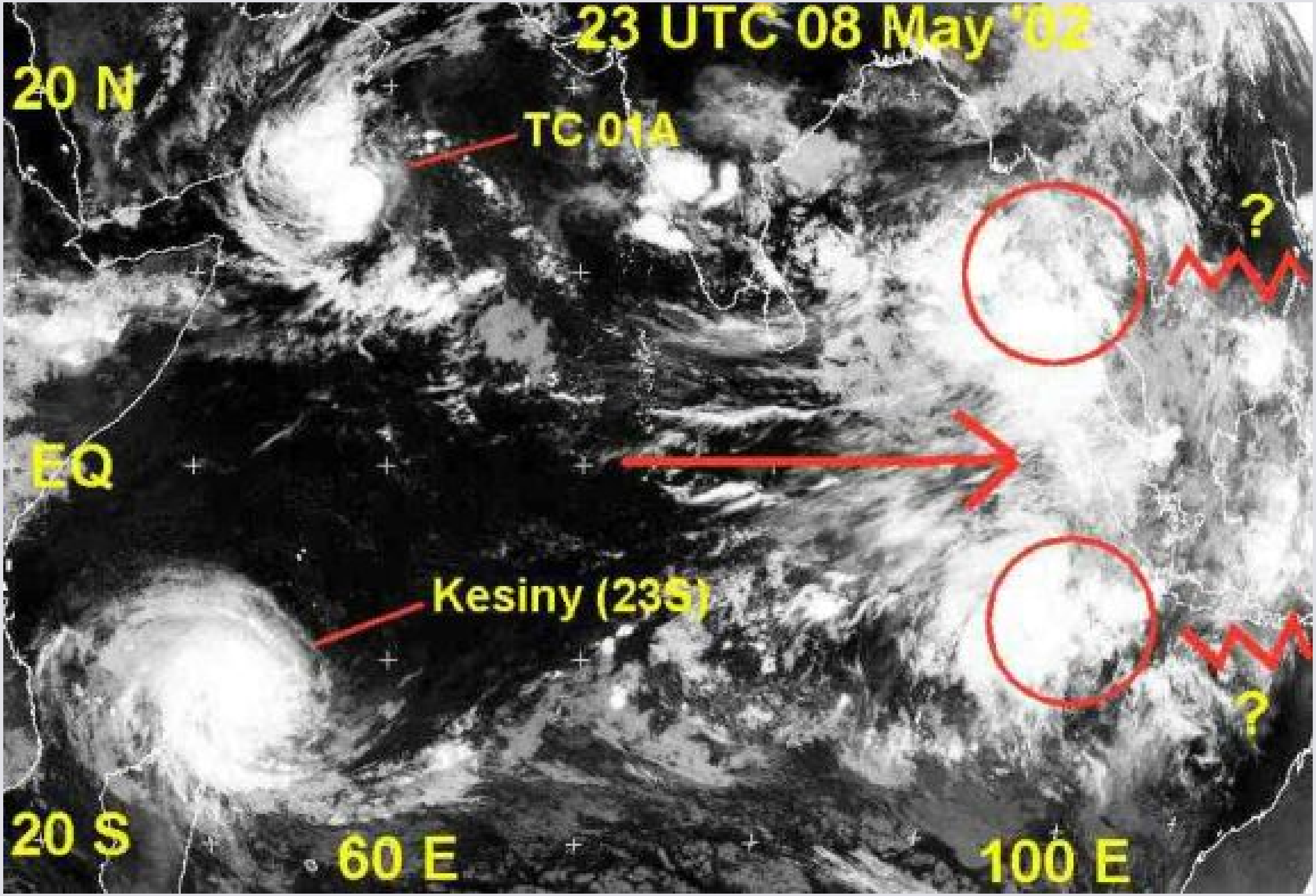
MJO in the Infrared: DYNAMO , Nov 4, 2011



MJO in visible light



MJO in the Infrared

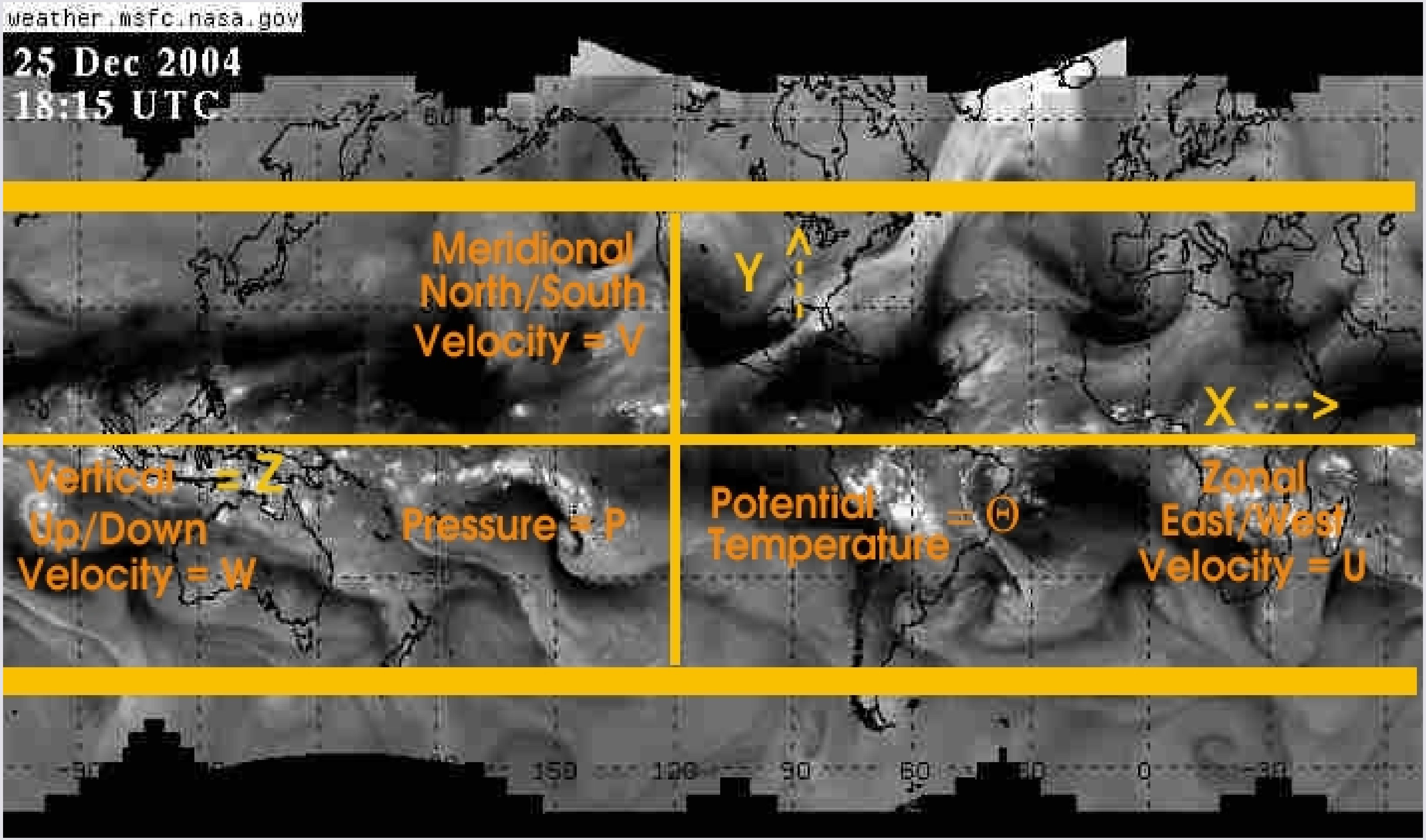


MJO in the Infrared

Overview of the Tropical Atmosphere

- Troposphere is extremely thin ($16 : 40000$ km)
- Length scales range from rain drop size to planetary scale.
- Time scales range from fractions of an hour to days to a few weeks (intraseasonal) to climate timescales.
- Coriolis parameter vanishes at the equator causing equatorial waveguide and unique nonlinear dynamics.
- Nonlinearities are strong on small scales, but weak on the largest scales.
- Forcing is dominated by latent heat release from water condensation when clouds are formed.

Coordinate System



The Equatorial Primitive Equations

Incompressible, Hydrostatic, Coriolis force on β-plane

$$
\begin{aligned}
u_t + \vec{u}\cdot\nabla\, u - \beta\, y\, v &= -p_x + S_u \\
v_t + \vec{u}\cdot\nabla\, v + \beta\, y\, u &= -p_y + S_v \\
\theta_t + \vec{u}\cdot\nabla\, \theta + N^2 w &= S_\theta \\
p_z &= \theta \\
u_x + v_y + w_z &= 0
\end{aligned}
$$

- $\vec{u} = (u, v, w) = (\mathrm{East}, \mathrm{North}, \mathrm{Up})$
- $\theta =$ potential temperature perturbation
- $p =$ pressure perturbation
- $N = \sqrt{\frac{g}{\theta_0}\frac{d\theta_0}{dz}} =$ buoyancy frequency
- $\beta\, y$ is vertical component of Coriolis force near equator
- Rigid lid $\Rightarrow w = 0$ at $z = 0,\ 16\,\mathrm{km}$

- Heat and momentum sources and sinks $= S_\theta,\ S_u,\ S_v$

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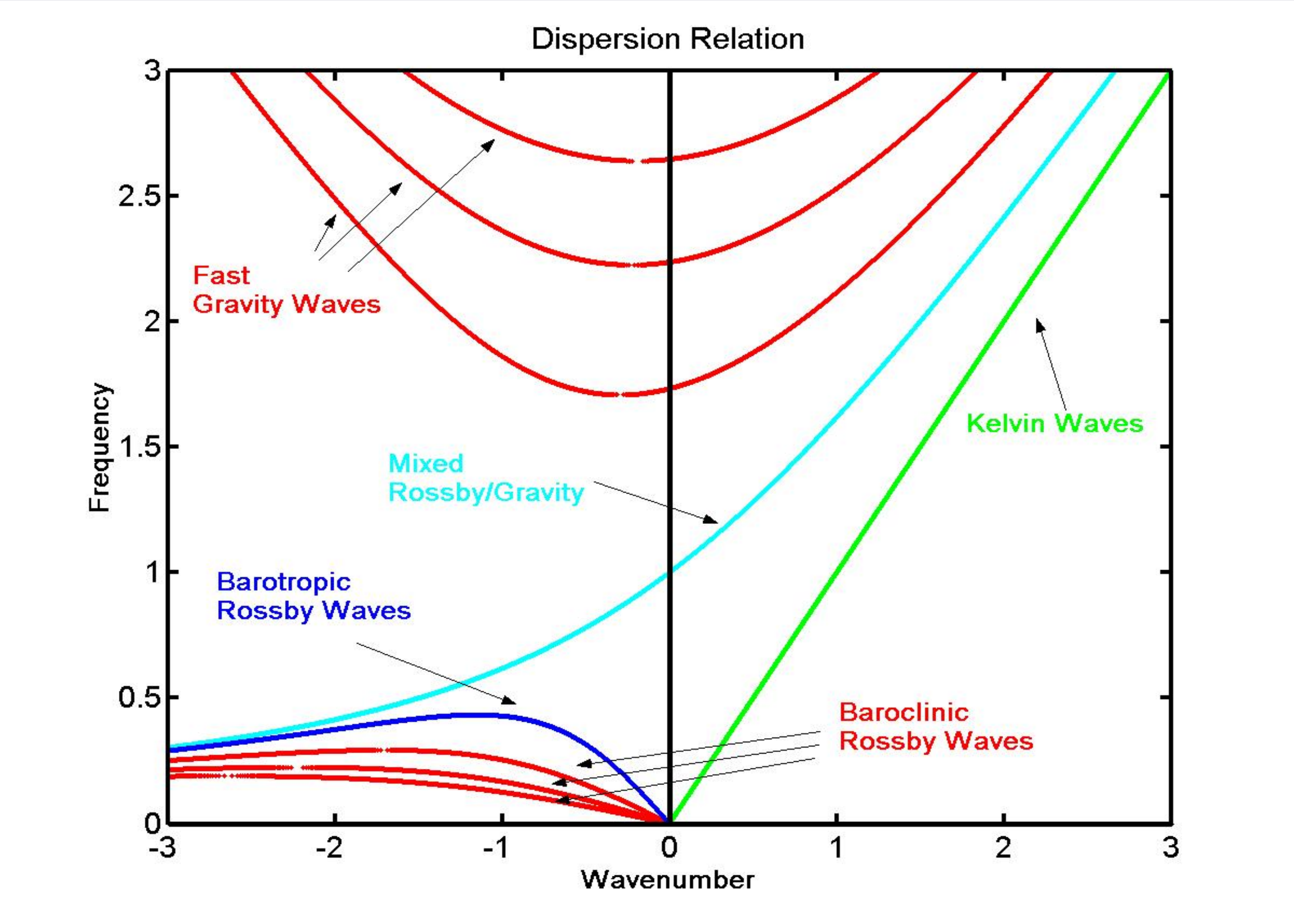
Schematically, these are equivalent to

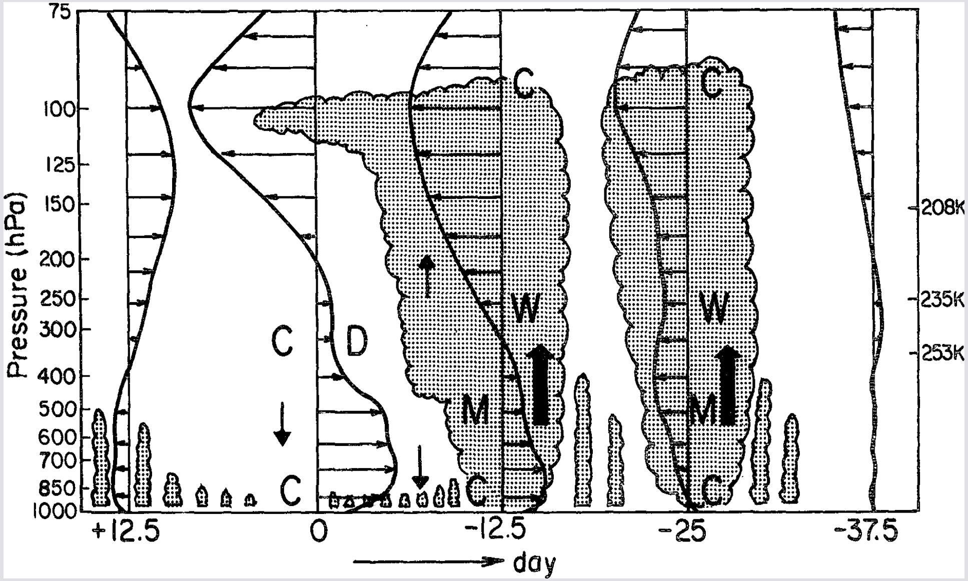
$$\mathbf{\Psi}_t + \mathbf{L}\,\mathbf{\Psi} + \mathbf{N}\,(\mathbf{\Psi},\ \mathbf{\Psi}) = \mathbf{S}(x, t)$$

Linear Theory of Equatorial Waves

$\mathbf{L}$ is skew self adjoint $\Longrightarrow$ dispersive waves



MJO: Vertical Shear and Convection



- Schematic showing correlation of convection and vertical shear
- Time goes from right to left (reverse of previous slide) and can be interpreted as left = west, right = east.

Lin & Johnson *J. Atmos. Sci.*, **53**, p 701, fig. 16.

- Congestus clouds - weak winds/easterlies
- Westward tilted anvil - westerly onset
- Strong westerlies trail convection

FRAMEWORK: Majda/Klein 2003, Biello/Majda 2010

Synoptic Scale (Balanced) Dynamics:

$$
\begin{aligned}
u'_\tau - y\,v' + p'_x &= S'_u \\
v'_\tau + y\,u' + p'_y &= S'_v \\
\theta'_\tau + w' &= S'_\theta \\
p'_z &= \theta' \\
u'_x + v'_y + w'_z &= 0 \\
\overline{S'_\theta} &= 0
\end{aligned}
$$

Planetary Scale Quasi-Linear Dynamics:

$$
\begin{aligned}
\overline{U}_t - y\overline{V} + \overline{P}_X &= F^U \quad -d_0\,\overline{U} \\
y\overline{U} + \overline{P}_y &= 0 \\
\overline{\Theta}_t + \overline{W} &= F^\theta \quad -d_\theta\,\overline{\Theta} + \overline{S}_\theta \\
\overline{P}_z &= \overline{\Theta} \\
\overline{U}_X + \overline{V}_y + \overline{W}_z &= 0
\end{aligned}
$$

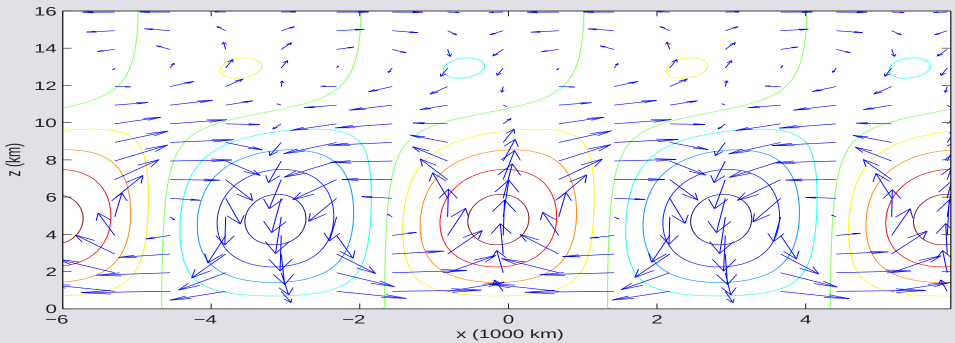
The fluxes from the synoptic scales are given by

$$
\begin{aligned}
F^U &= -\overline{(v'\,u')_y} - \overline{(w'\,u')_z} \\
F^\theta &= -\overline{(v'\,\theta')_y} - \overline{(w'\,\theta')_z}
\end{aligned}
$$

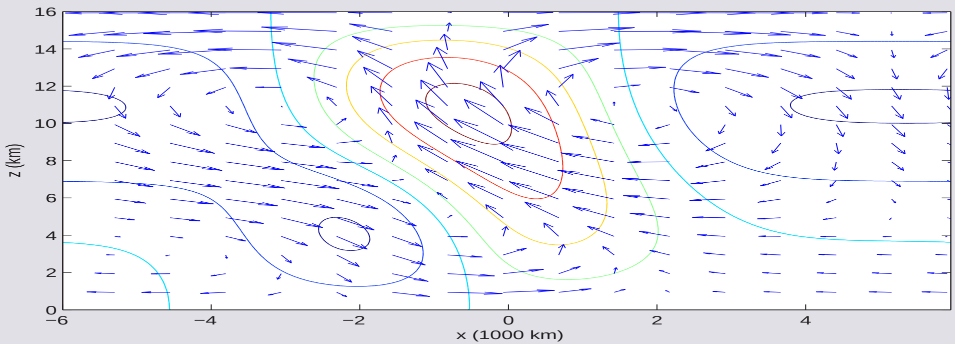
Each forcing effect, i.e. upscale vertical and meridional momentum and temperature transport and planetary scale mean heating can be considered separately and superposed

MJO Model: Convection organized on small scales

- Heating rate traces cloudiness (latent heat release).
- Fluctuations on 1500 km spatial scales
- Clouds/heating localized near equator above Western Pacific.

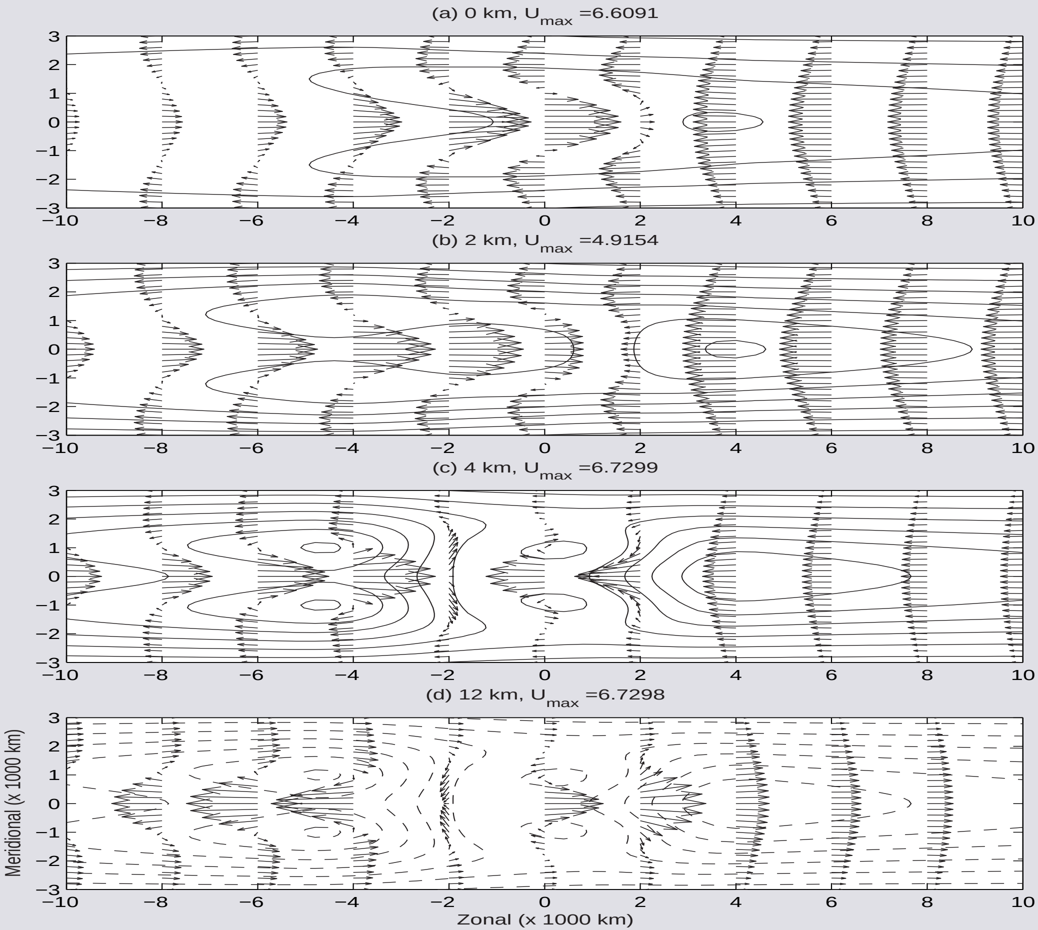


- East: Lower troposphere ***congestus clouds***
- West: High, westward tilted anvil ***superclusters***
- Flow vectors and heating contours
- Upscale flux, $\overline{\mathbf{N}\,(\psi',\,\psi')} \neq 0$
  $\Rightarrow$ Vertical/Longitudinal Tilt



Equatorial MJO model: Flow in the Horizontal Plane

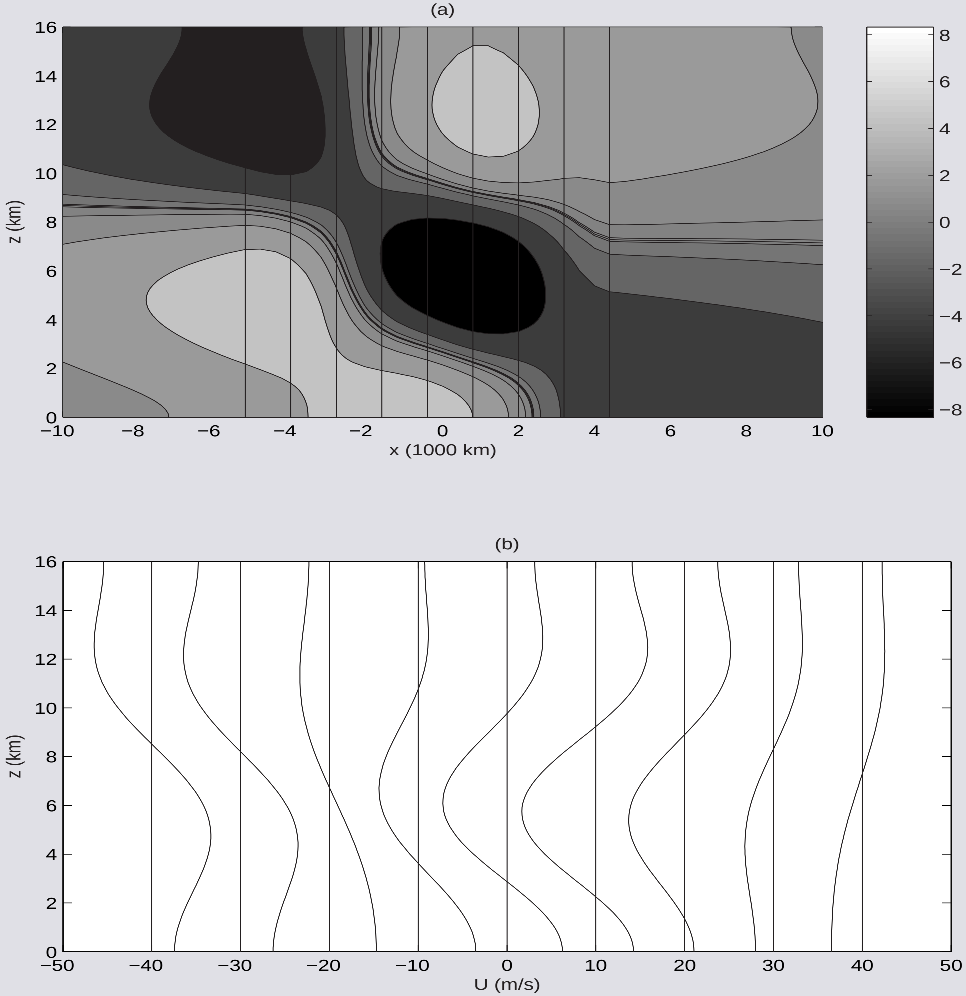
- Congestus heating in the east and westward tilted superclusters in the west of a moving warm pool.
- Planetary mean heating is weaker, but has same structure of synoptic scale fluctuations.
- Pressure and flow at $z = 0, 2, 4, 12$ km.



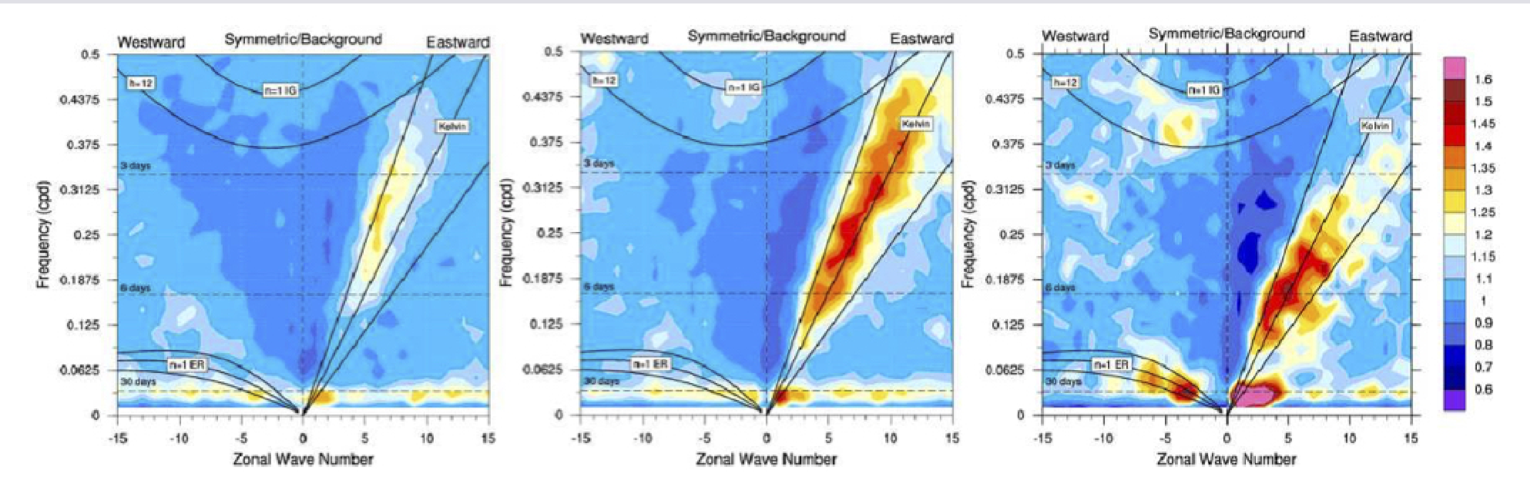
***

Equatorial MJO model: Winds above the equator

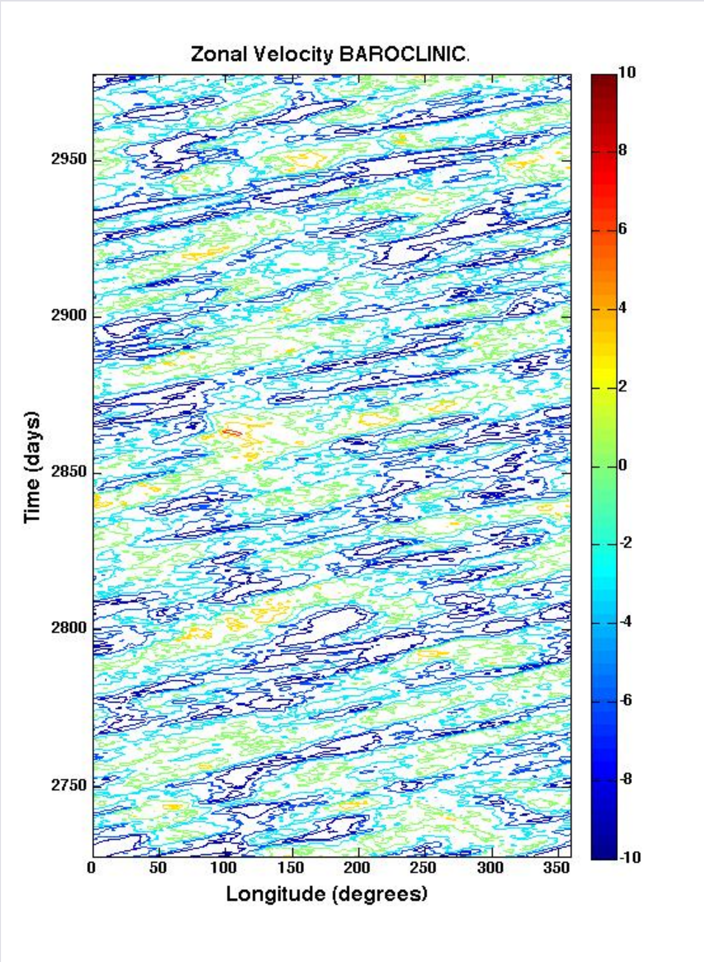
- Lower troposphere congestus heating in the east
- Westward tilted anvil superclusters in the west
- (a) Zonal velocity:
  westerly = light,
  easterly = dark
  versus height and longitude above equator
- (b) Height vs Velocity.



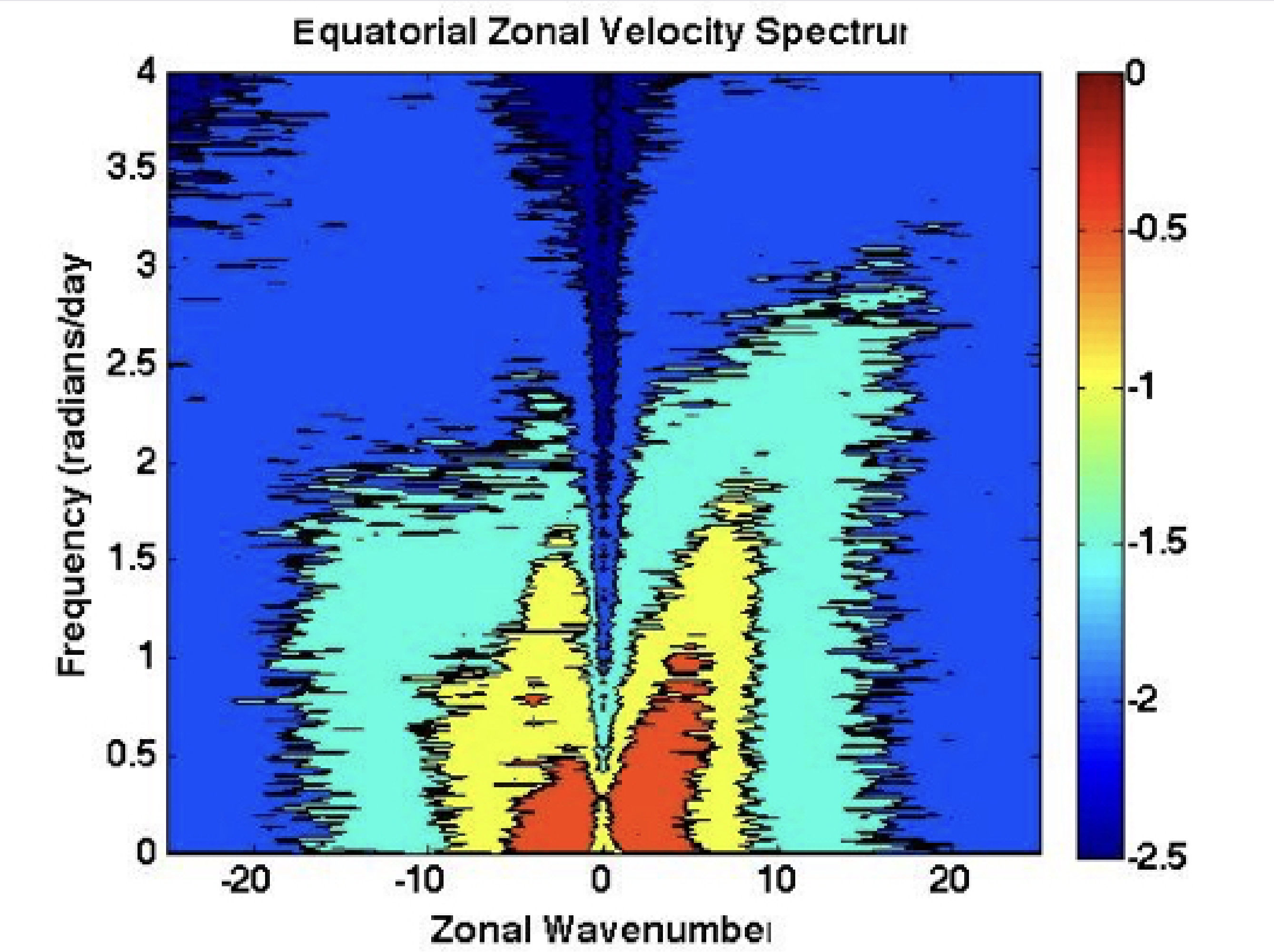
Spectral features of computation and data



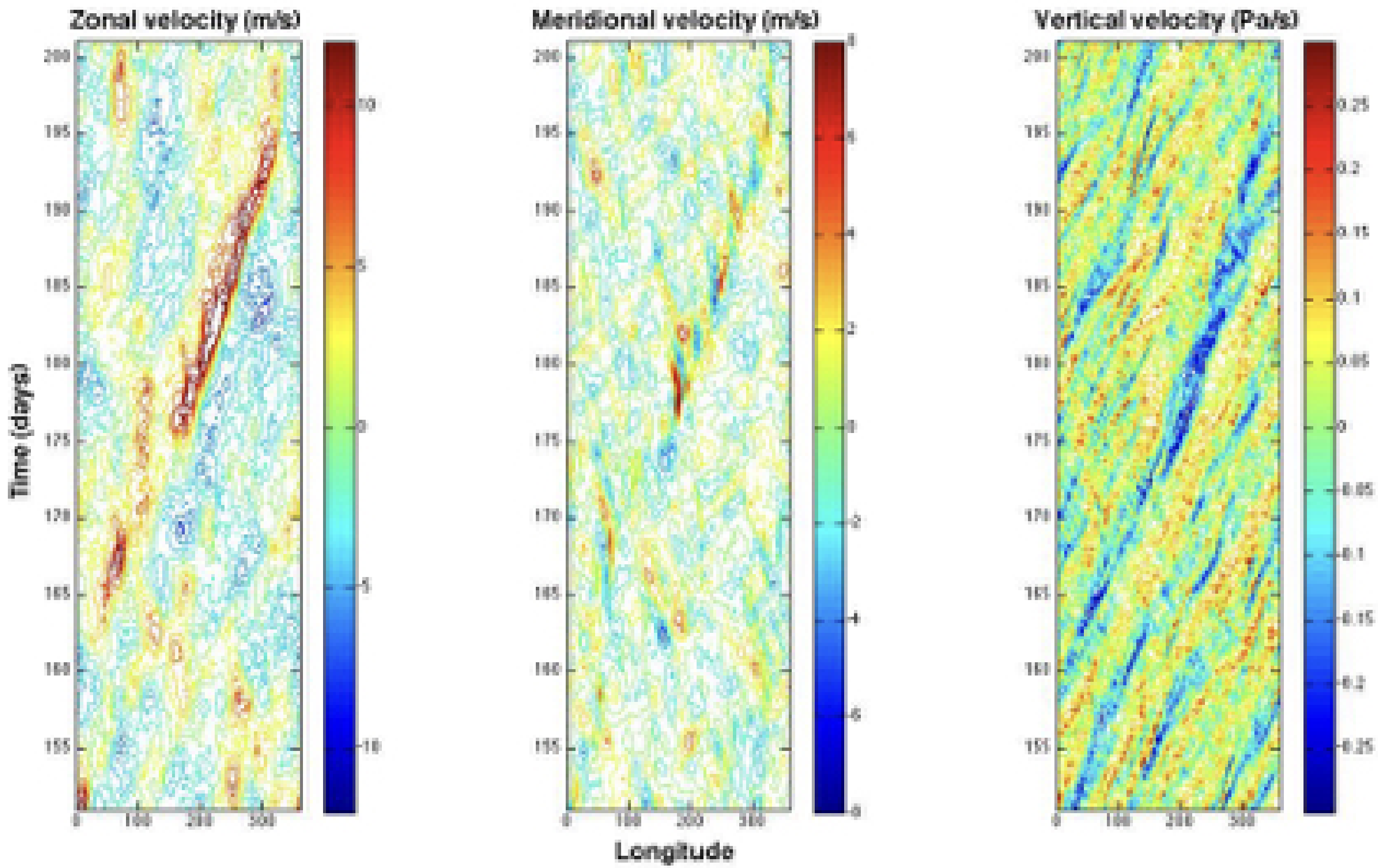
Space time diagram of ECMWF results - Kelvin Waves



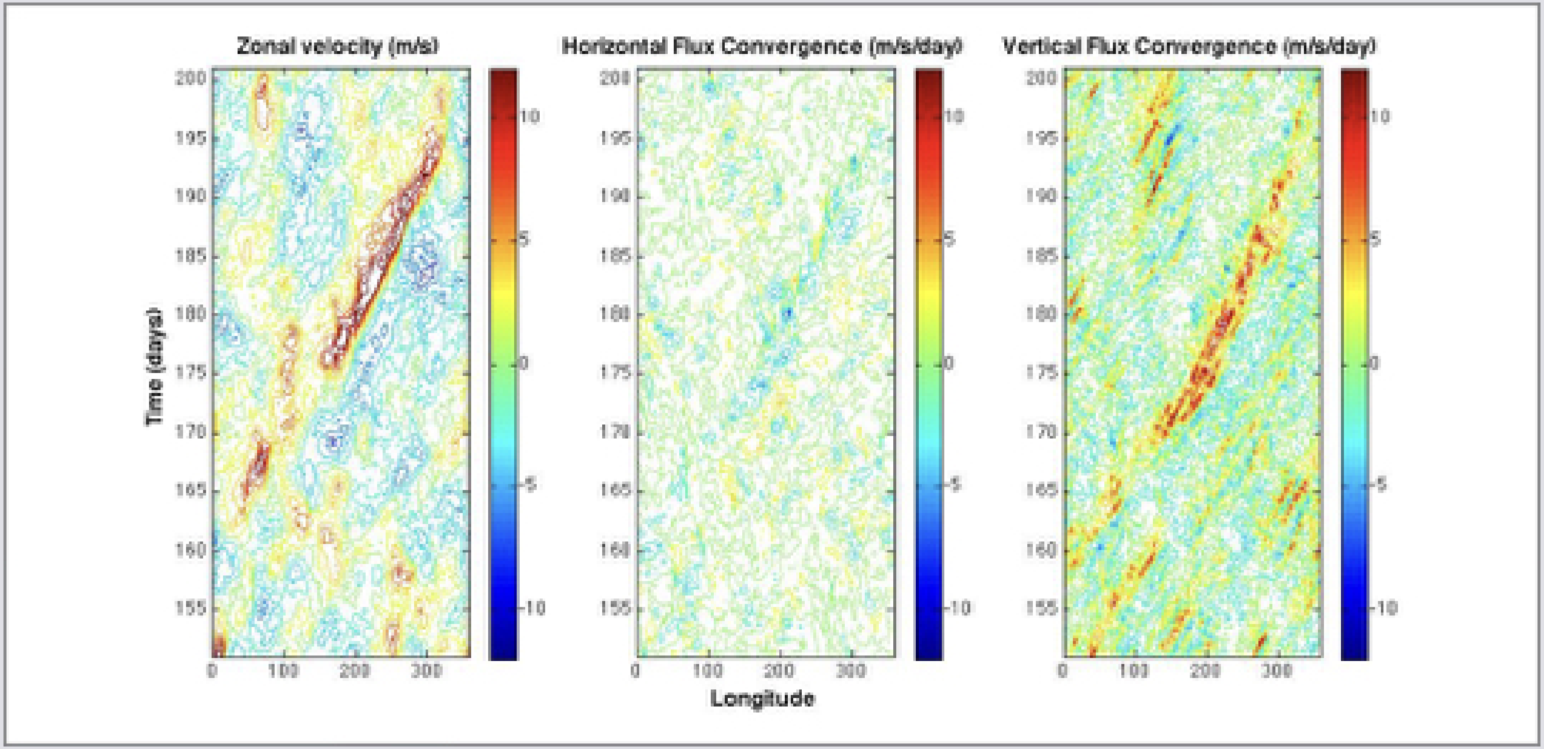
Spectrum of ECMWF results



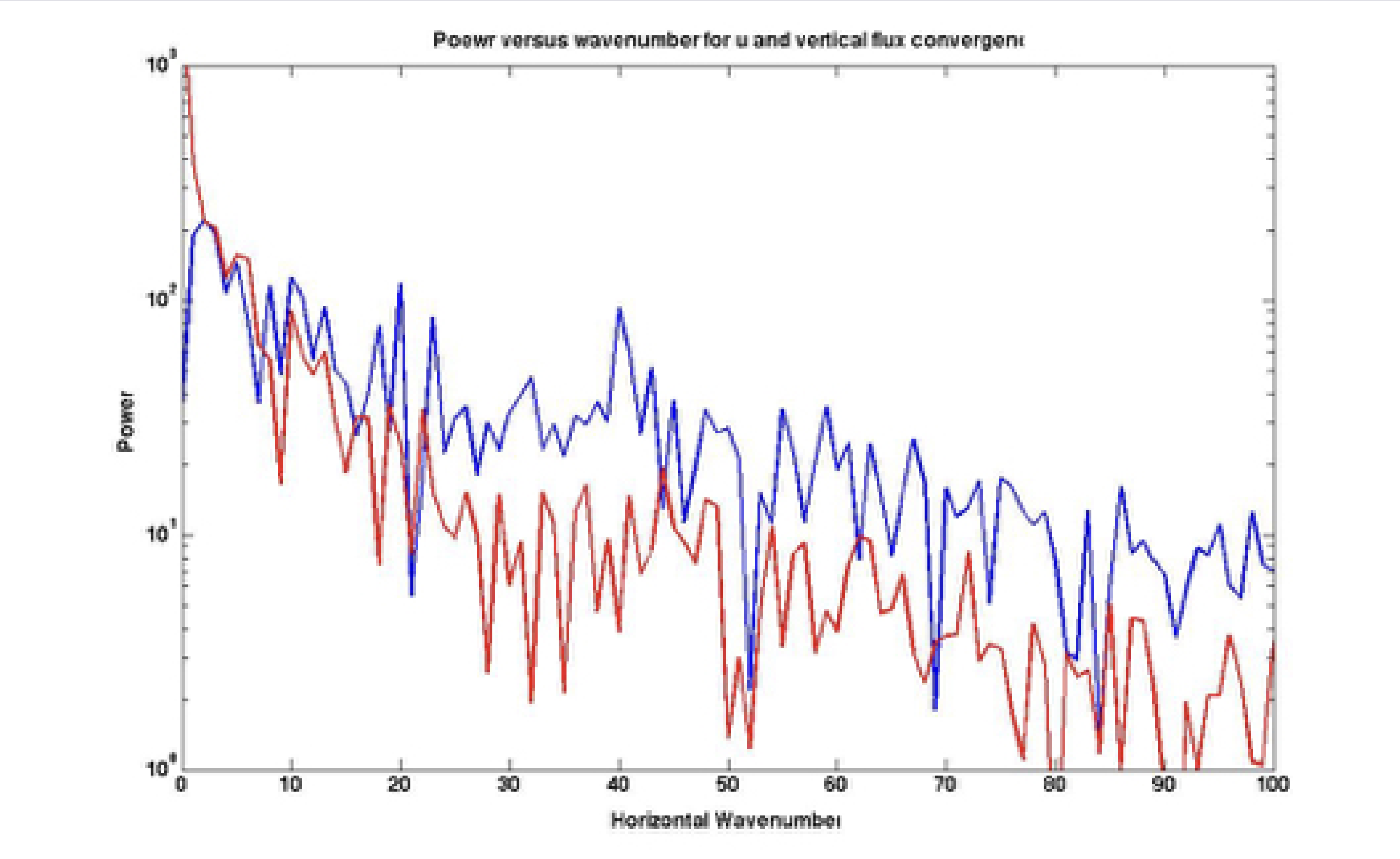
Space time diagram of ECMWF results - Westerly Wind Burst



Space time diagram of ECMWF results - Westerly Wind Burst



Fluxes from ECMWF results



In Conclusion

- Numerical simulations are still unable to simulate large scale organized convection.
- I believe that this is due to the inability of the convective models to generate vertical and meridionally tilted structures.

$$
\begin{aligned}
F^U &= -\overline{(v'\,u')_y} - \overline{(w'\,u')_z} \\
F^\theta &= -\overline{(v'\,\theta')_y} - \overline{(w'\,\theta')_z}
\end{aligned}
$$