Flagellar swimming in viscoelastic fluids: Role of fluid elastic stress revealed by simulations based on experimental data

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joint work with Chuanbin Li, Boyang Qin, Arvind Gopinath,

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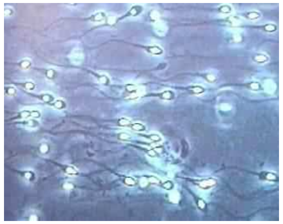
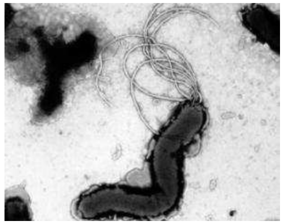
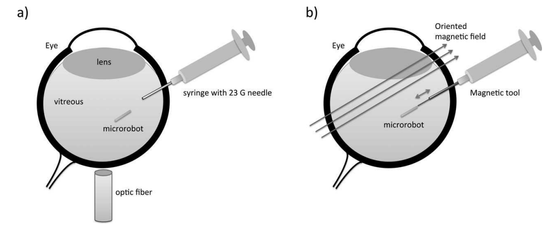
Locomotion and Transport in Complex Fluids







- Complex fluids, such as mucus, are mixtures of water and polymer
- Historically, studies done in Newtonian fluid (i.e. water)
- Many recent studies on locomotion in viscoelastic fluids
- However, still missing a mechanistic understanding of how fluid rheology affects locomotion

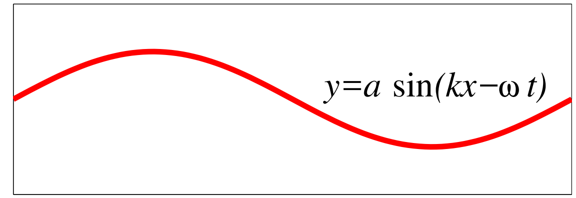
Classical swimming sheet problem

First analyzed by G.I. Taylor (1951)

Solve

$$\mu \Delta u - \nabla p = 0$$

$$\nabla \cdot u = 0$$



in swimmer's reference frame

subject to $u = (0, \dot{y})$ on boundary $y = a\sin(kx - \omega t)$.

in the limit $\epsilon = ak \to 0$ using asymptotic expansion.

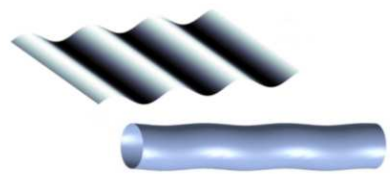
Swimming speed is $U_0 = -u(\infty)$

$$U_0 = \frac{1}{2}\omega k a^2 + \mathcal{O}(\epsilon^4)$$

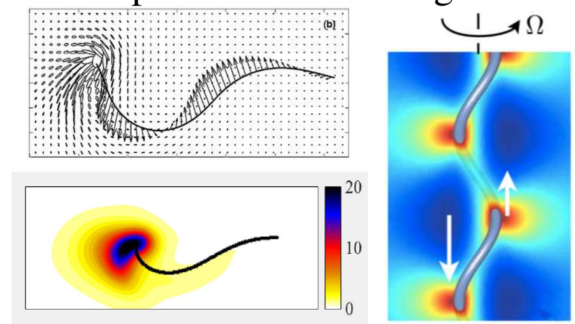
<u>Viscoelastic version analyzed over 50 years later, Lauga (2007)!</u>

Different Approaches, Different Conclusions

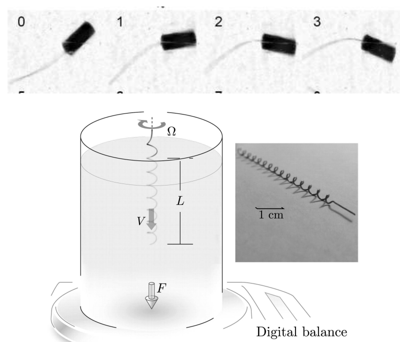
Small Amplitude Analysis



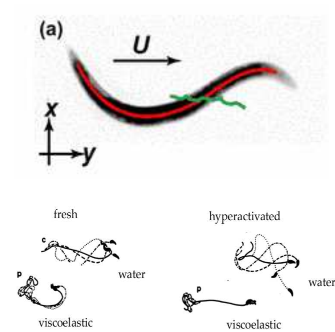
Computational Modeling



Physical Models



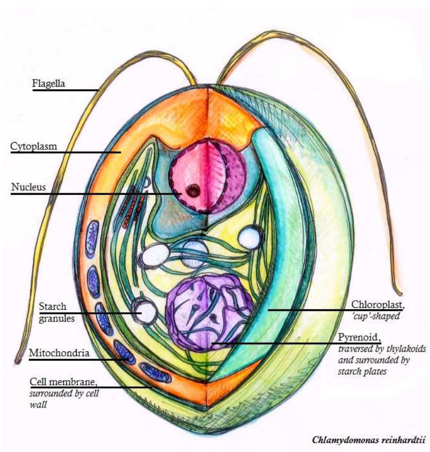
Biological Experiments

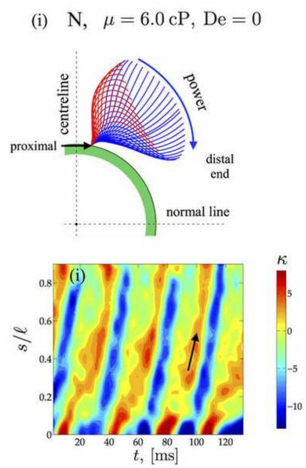


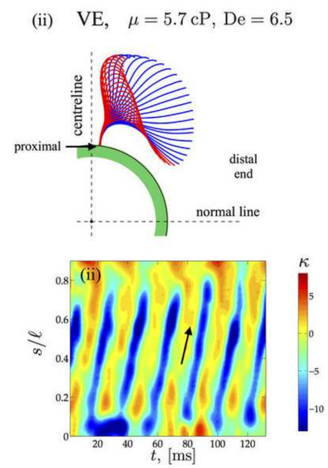
- Conflicting results between experiments, physical models, analysis, and computations
- This problem is <u>hard</u>: nonlinear, time dependent, model/fluid/gait dependent
- We are after the basic principles of how elastic stress affects swimming
  <u>This talk</u>: develop computational model in conjunction with experimental data
  Li et al., Royal Society Interface (2017)

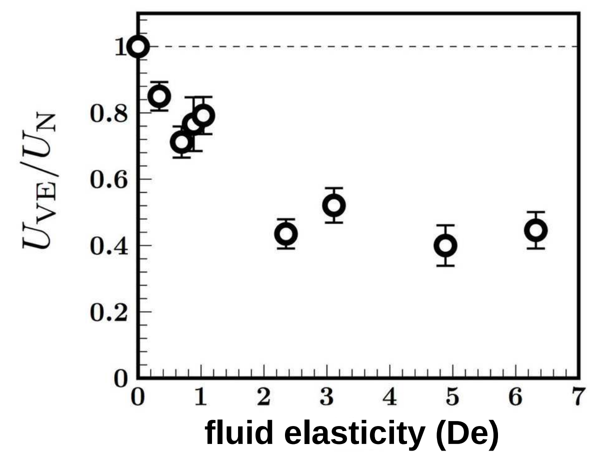
Experiments

Flagellar Kinematics and Swimming of Algal Cells in Viscoelastic Fluids, Qin et al. (2015)









- Measured how flagellar stroke changes with both viscosity and elasticity
- Experimental measurements of swimming as a function of rheology
- Experiments alone cannot be used to understand effect of elasticity

Here we use computational model to change stroke and rheology independently

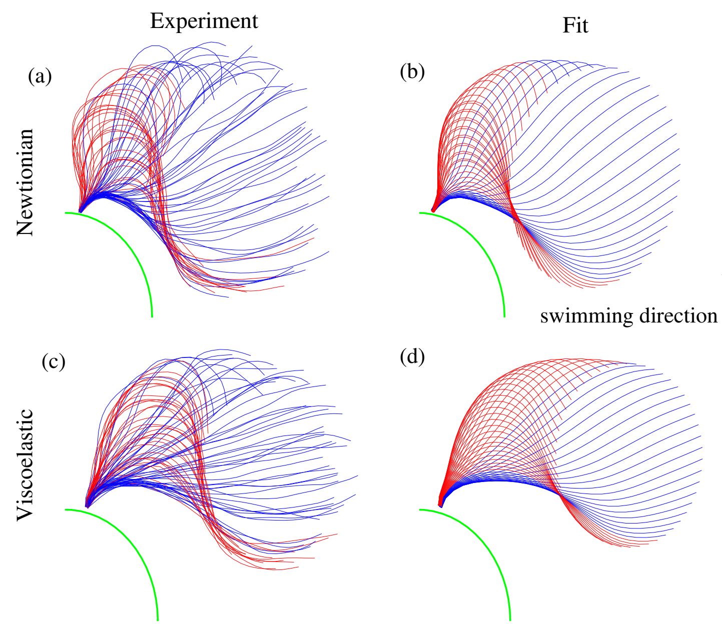
Qin Movie Leptos Movie

Model Strokes

Experimental data fit with model stroke of the form

$$X_i(s,t) = M_i(s) + A_i(s)\cos\big(2\pi\omega t + \phi_i(s)\big), \quad i = 1,2$$

$$X_3(s,t) = 0$$



Model "Newtonian stroke" from fluid with $\mu = 2.6$ cP

Model "viscoelastic stroke" from fluid with $\mu = 2.5$ cP, De=2

Prescribed Shapes

$\mathbf{X}_p(\mathbf{s}, t)$: prescribed shape of swimmer in fixed body frame

$\mathbf{X}_0(t)$: translation of body from to lab frame

$\mathbf{X}(s, t)$: shape of swimmer in the lab frame

$$
\begin{aligned}
\mathbf{X}(s,t) &= \mathbf{X}_p(s,t) + \mathbf{X}_0(t) \\
\partial_t \mathbf{X}(\mathbf{s},t) &= \mathbf{U}(\mathbf{s},t) \\
\mathbf{U}(s,t) &= \mathbf{U}_p(s,t) + \mathbf{U}_0(t)
\end{aligned}
$$

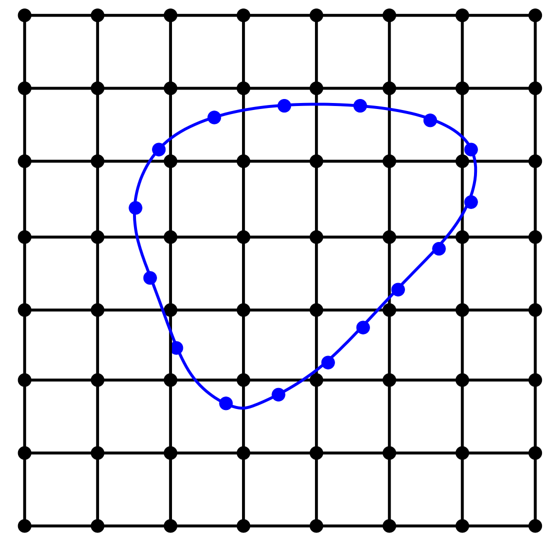
- Velocity in body frame ($\mathbf{U}_p$) prescribed, surface force density ($\mathbf{F}$) is unknown
- Translational velocity ($\mathbf{U}_0$) determined by no net force constraint
- Body modeled as ellipsoid, diameters 10 μm, 10 μm, 12 μm.

IB Method – Mixed Eulerian/Lagrangian Formulation

- Use ideas of Immersed Boundary Method to couple fluid and structure
- Structure represented in Lagrangian coordinate
  $\mathbf{X}(\mathbf{s},t)$ position
  $\mathbf{U}(\mathbf{s},t)$ velocity
  $\mathbf{F}(\mathbf{s},t)$ force density
- Fluid/stress represented in Eulerian coordinates
  $\mathbf{u}(\mathbf{x},t)$ velocity
  $p(\mathbf{x},t)$ pressure
  $\boldsymbol{\tau}(\mathbf{x},t)$ polymer stress
- Eulerian and Lagrangian variables related by

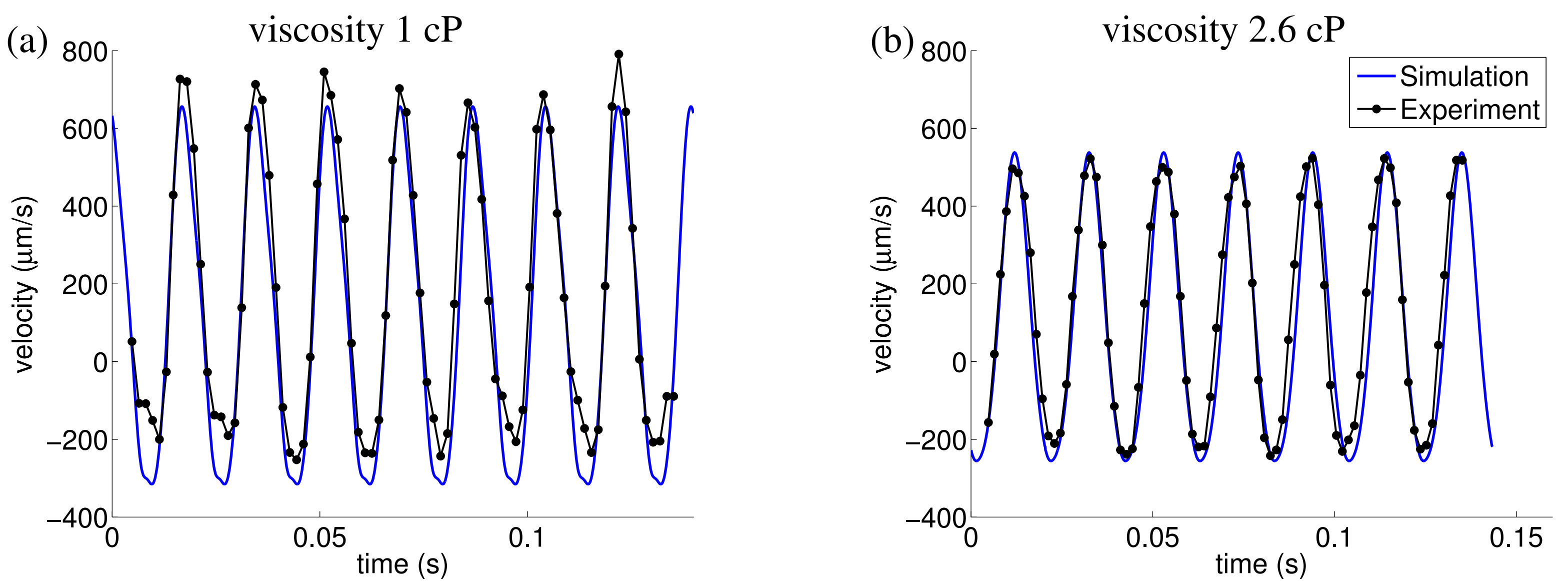
$$\mathbf{f}(\mathbf{x},t) = \mathcal{S}\mathbf{F} = \int_{\text{swimmer}} \mathbf{F}(\mathbf{s},t)\delta\left(\mathbf{x}-\mathbf{X}(\mathbf{s},t)\right)\,d\mathbf{s},$$

$$\mathbf{U}(\mathbf{s},t) = \mathcal{S}^*\mathbf{u} = \int_{\text{fluid}} \mathbf{u}(\mathbf{x},t)\delta\left(\mathbf{x}-\mathbf{X}(\mathbf{s},t)\right)\,d\mathbf{x}.$$



Results for Newtonian Fluid – Model/fit Validation

Model consistent with the experimental data on swimming speed

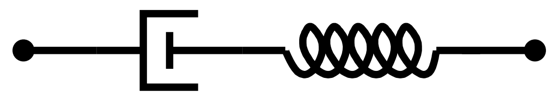


Oldroyd-B Model for Viscoelasticity

$$\tau_{\text{total}} = \tau_{\text{fluid}} + \tau_{\text{polymer}}$$

Maxwell model

$$\lambda\dot{\tau} + \tau = 2\mu_{\text{p}} D$$



λ is the relaxation time

μ_p is the polymer viscosity

D is deformation rate tensor

Being careful with time derivative – upper convected Maxwell equation**

$$\lambda(\tau_t + u \cdot \nabla\tau - \nabla u\tau - \tau\nabla u^{\text{T}}) + \tau = 2\mu_p D$$

Deborah Number: $De = \dfrac{\text{relaxation time}}{\text{flow time scale}}$

$De \to 0$ recover Newtonian fluid

$De \to \infty$ recover neo-Hookean elastic solid

De is a measure of the elasticity of the fluid

**Can also be derived from dilute suspension of dumbbells connected by linear springs

Model Equations and Method

Fluid Equations

$$\mu_s \Delta \mathbf{u} - \nabla p + \nabla \cdot \boldsymbol{\tau} + \mathcal{S}\mathbf{F} = 0$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\lambda \left(\boldsymbol{\tau}_t + \mathbf{u} \cdot \nabla \boldsymbol{\tau} - \nabla \mathbf{u}\, \boldsymbol{\tau} - \boldsymbol{\tau} \nabla \mathbf{u}^{\mathrm{T}} \right) + \boldsymbol{\tau} = 2\mu_p \mathbf{D} + \epsilon \Delta \boldsymbol{\tau}$$

Swimmer Constraints – determine swimmer forces and translational velocity implicitly

$$\mathcal{S}^* \mathbf{u} = \mathbf{U}_p + \mathbf{U}_0$$

$$\int_{\text{swimmer}} \mathbf{F}\, d\mathbf{s} = 0$$

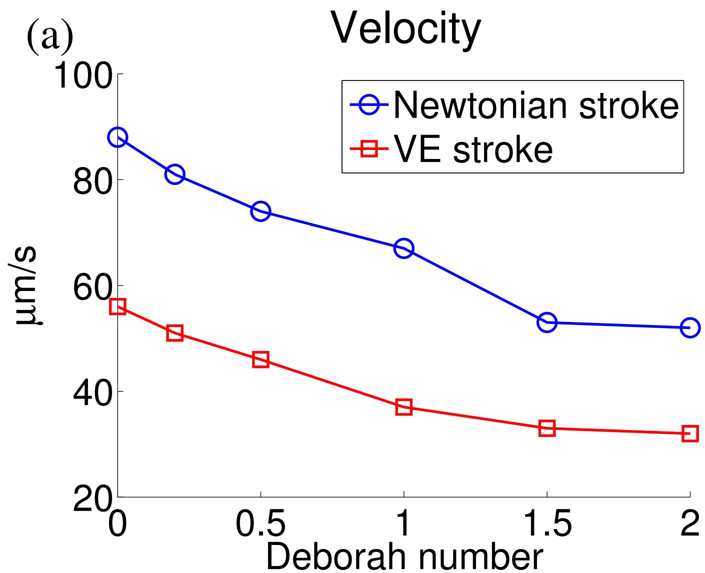
Some Numerical Details

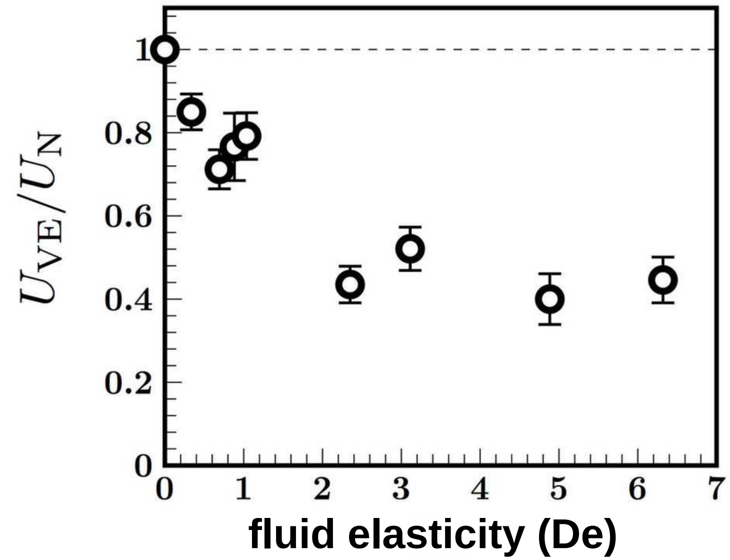
- Alternate time updates of VE stress and fluid/swimmer system
- Pseudospectral method with $\Delta x \approx 370$ nm (close to flagella thickness)
- Given VE stress solve first for swimmer force and translation using Schur complement solved by PCG

Outline for Remainder of Talk

1. Separating the effects of rheology and stroke
   (a) Compare performance of “Newtonian stroke” and “viscoelastic stroke” in fluids of different relaxation times.
   (b) Use the “vicoelastic stroke” in fluids with different relaxation times (Deborah numbers) to explore effects of fluid elasticity independent of the stroke.
2. A mechanistic explanation of elastic effects on swimming
   (a) Quantify effects of elastic stress on swimming performance
   (b) Explain observations of stress development and relate to stroke

Fix the Stroke, Change the Fluid

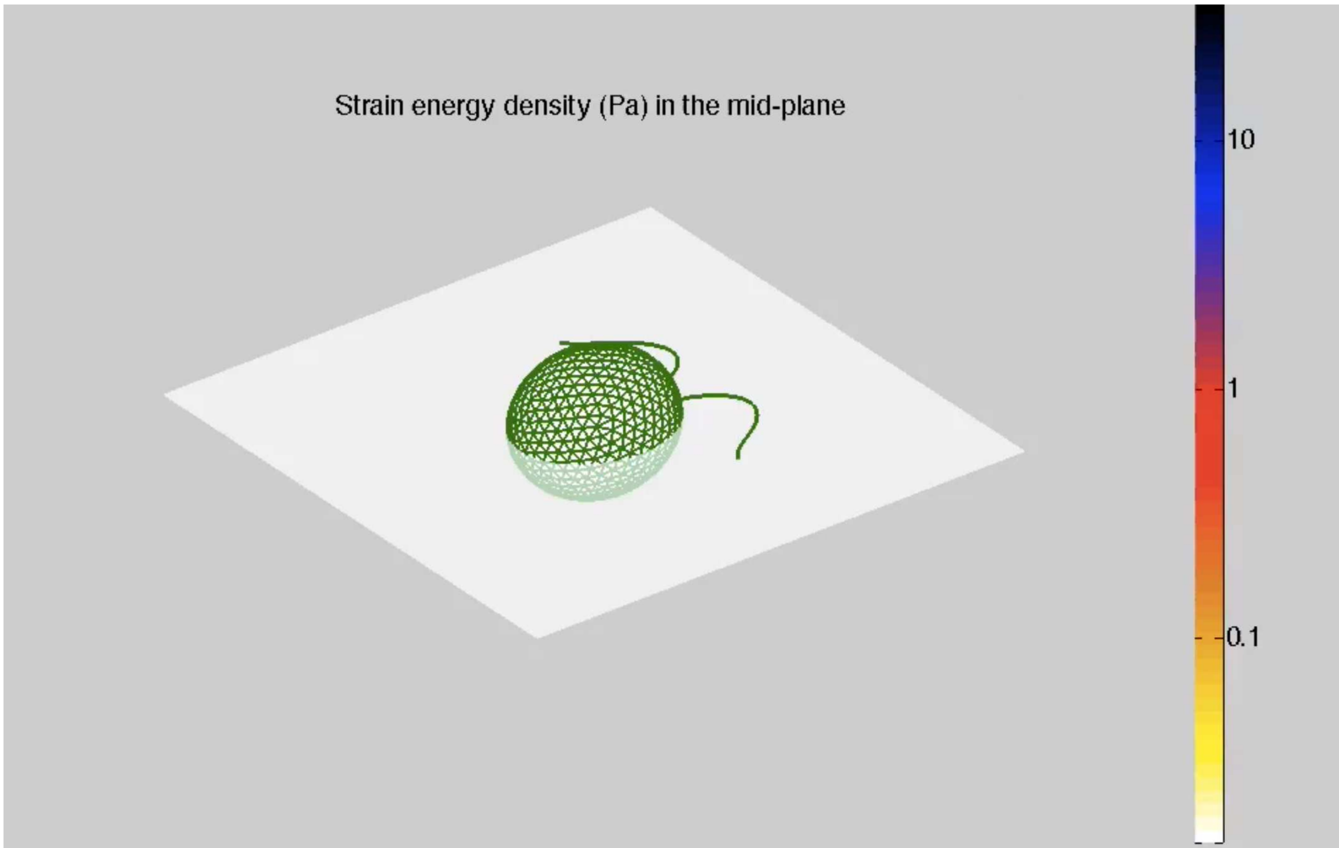




- Newtonian stroke swimming speeds faster than viscoelastic stroke
- Model results consistent with experiments:
  VE stroke 0.4 times slower in VE fluid compared to Newtonian stroke in Newtonian fluid.
- NOT SHOWN: Stress and power consumption much greater for Newtonian stroke.

Visualizing the Stress

VE stroke in fluid with $De = 2$

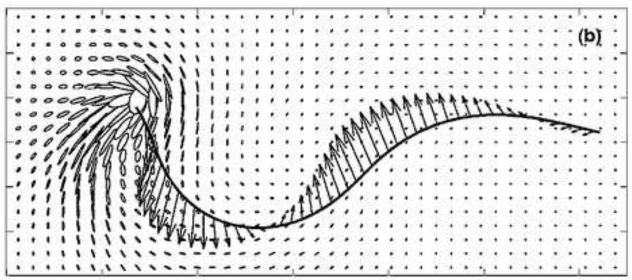


Movie

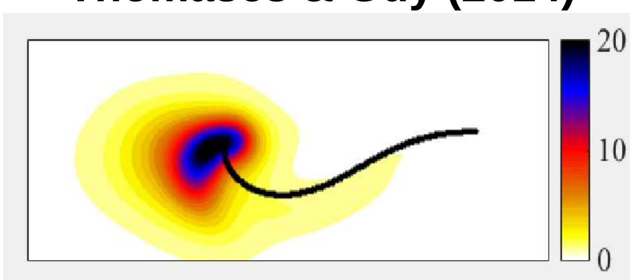
Big Tip Stresses

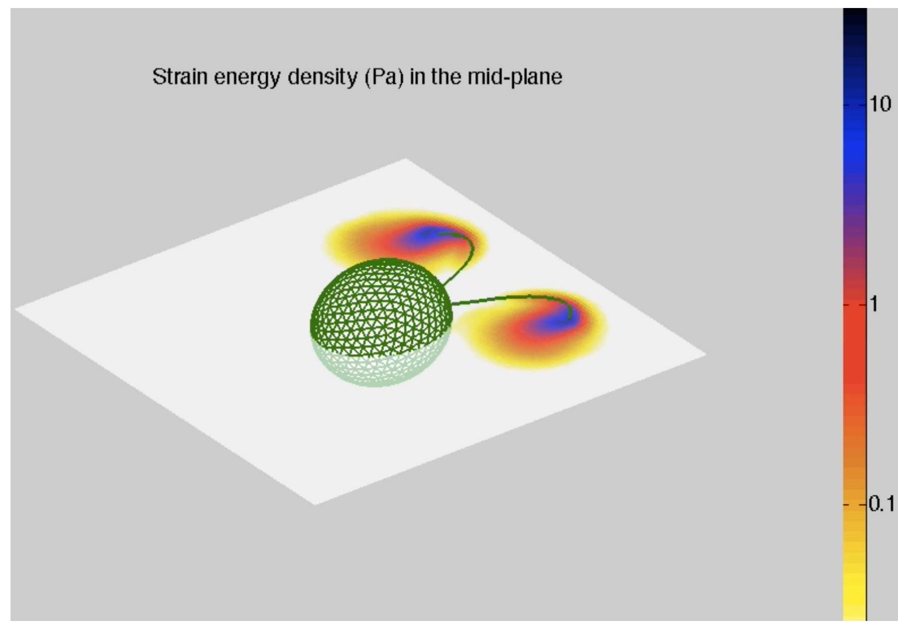
Large elastic tip stresses have been seen in previous 2D in computations

Teran et al. (2010)



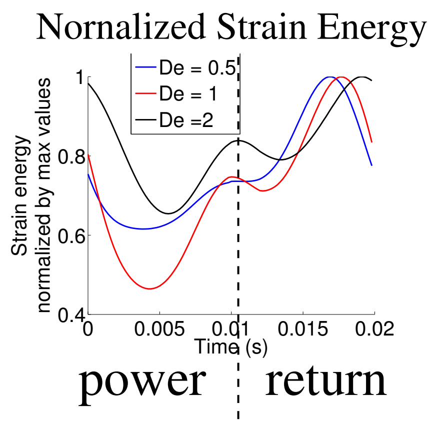
Thomases & Guy (2014)

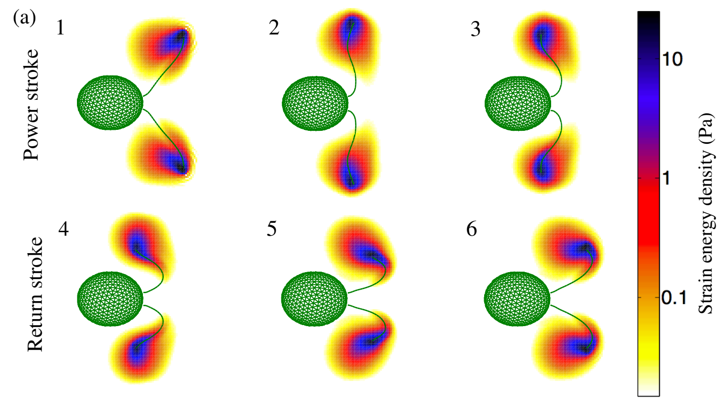


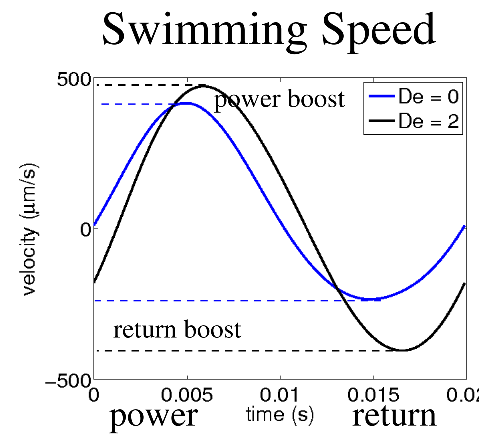


These are the first 3D computations in VE using biological gait

Stress and Speed in Time







- Stress is generally decreases during the power stroke and increase during the return stroke.
- Speed on both power stroke and return stroke increases with De
- Speed increase is larger on return stroke than on power stroke

Some Key Questions

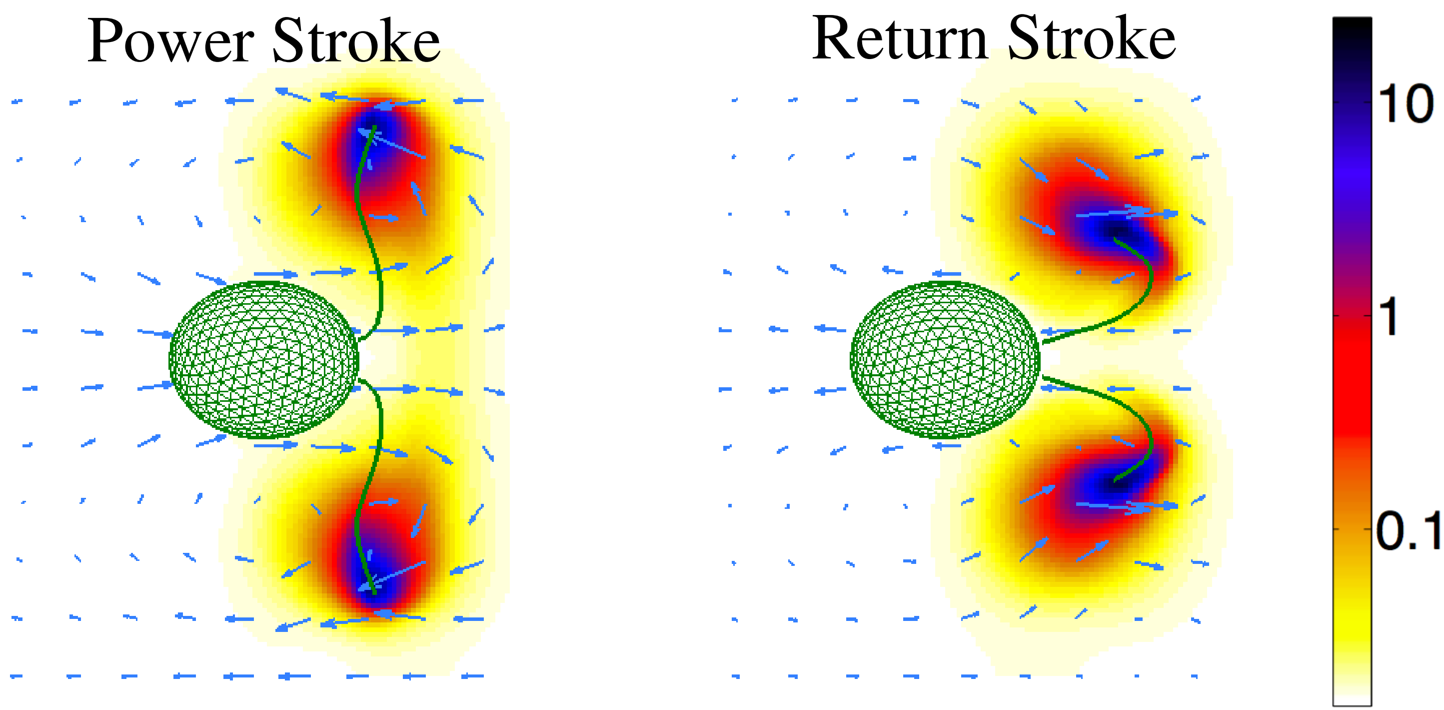
1. Do these observations depend on the stroke?
   No, but I didn't show you the data

2. How are the accumulated stresses related to speed enhancements?

3. Why is there an asymmetric stress response on power and return?

Effect of Accumulated Elastic Stress

Suddenly fix swimmer shape, accumulated stress drives a flow

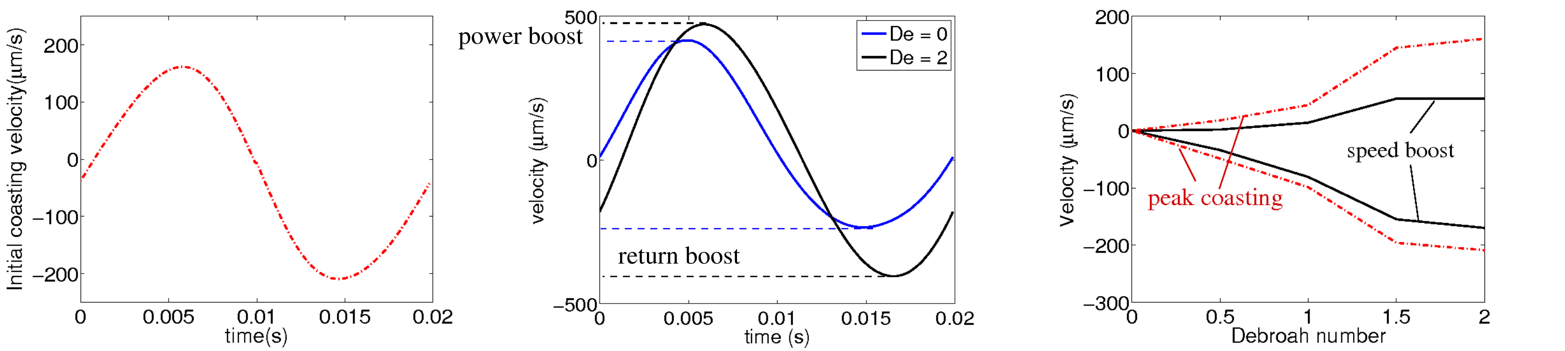


The swimmer continues to move in the direction it was traveling when the stroke was frozen – reversed from intuition (expect elastic recoil)

Initial Coasting Velocity

Initial Coasting Velocity – initial velocity of the swimmer after the stroke is frozen

Initial Coasting Velocity Swimming Speed Speed Boost and Peak Coasting

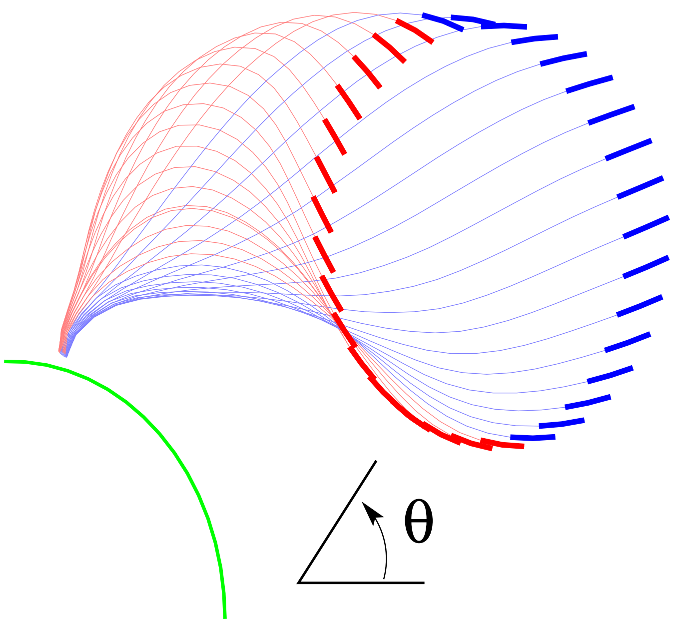


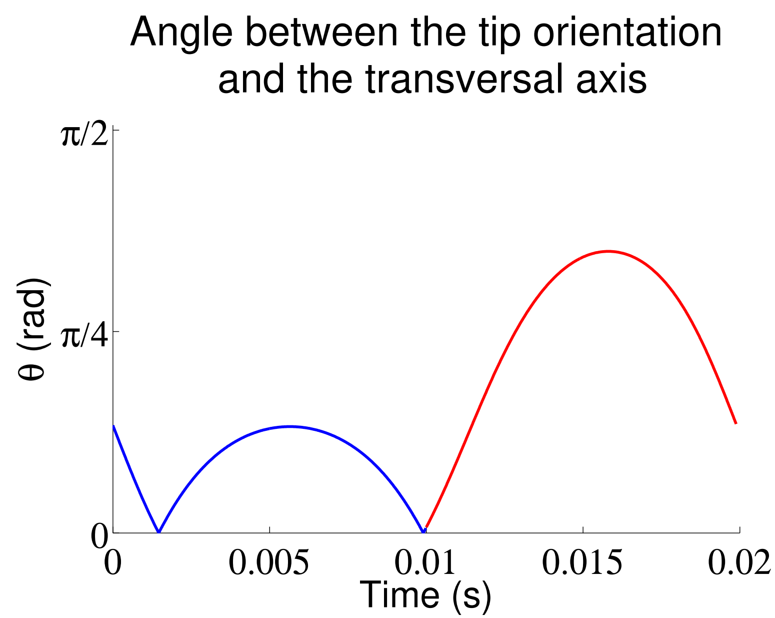
- speed boost and initial coasting velocity show the same trend
- swimmer continues in same direction after stroke changes direction – effective “elastic inertia”

Question: Why are stresses, speed boost, and coasting velocity larger on return stroke?

Tip Orientation on Power and Return

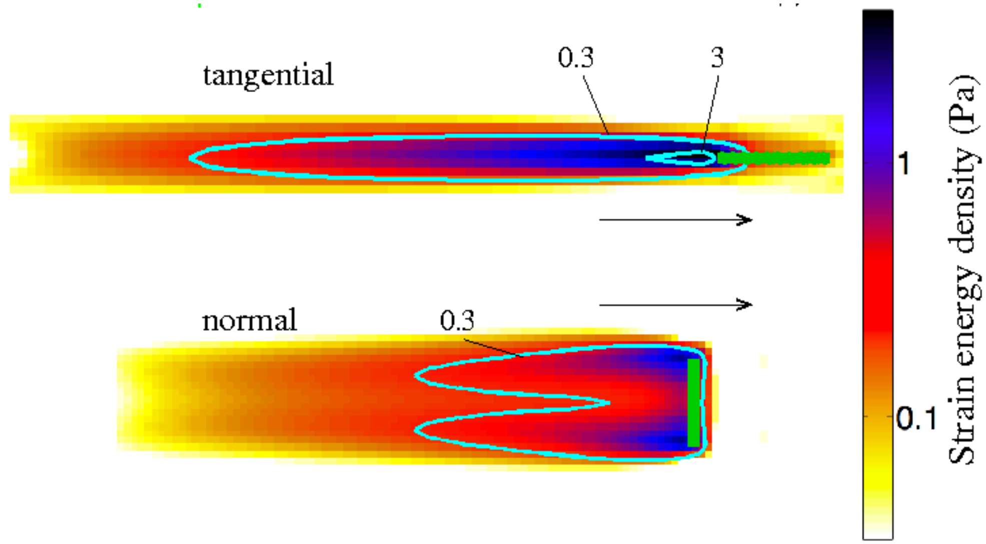
Tip is more aligned with the swimming direction on return stroke

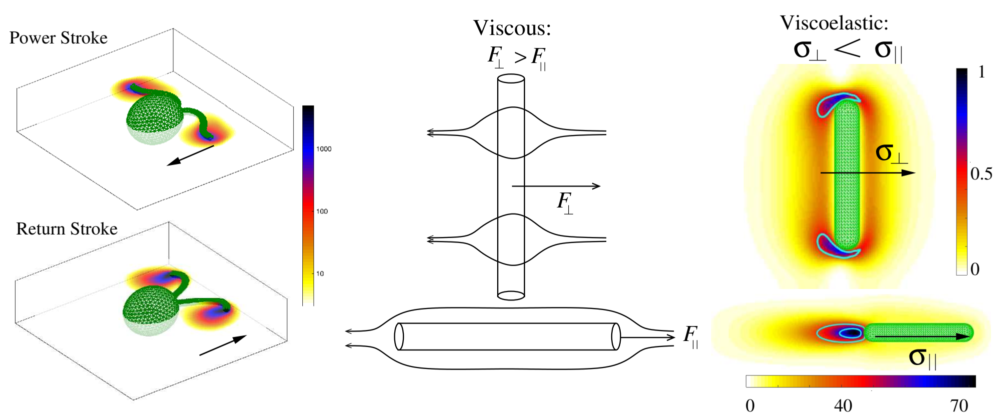




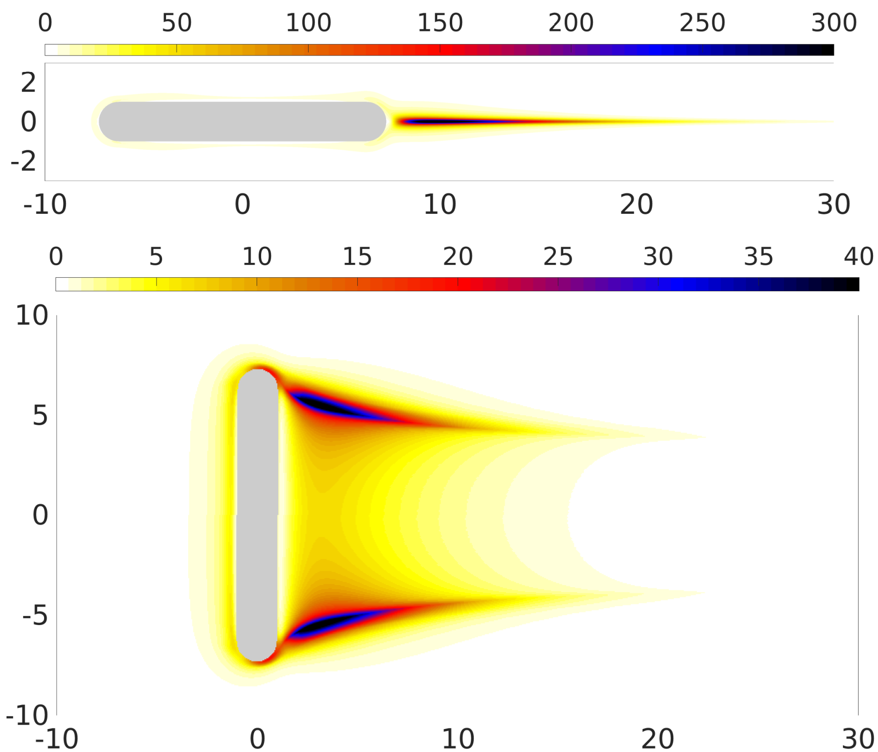
Rod Moving with Different Orientations

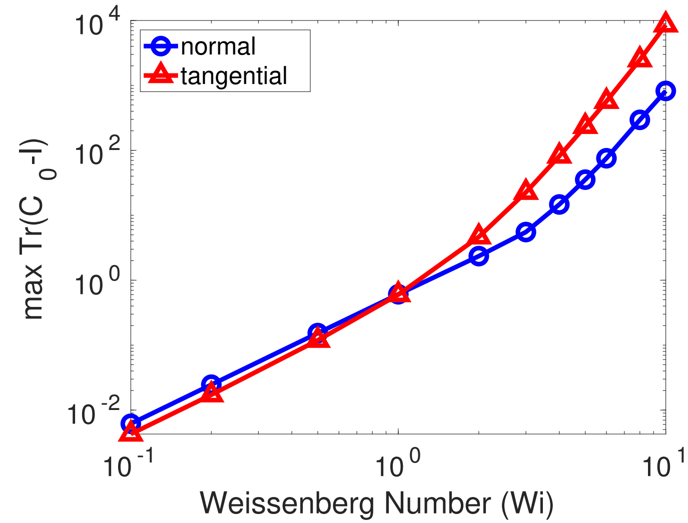
Elastic stress is larger for tangential motion





Rods Moving with Different Orientations

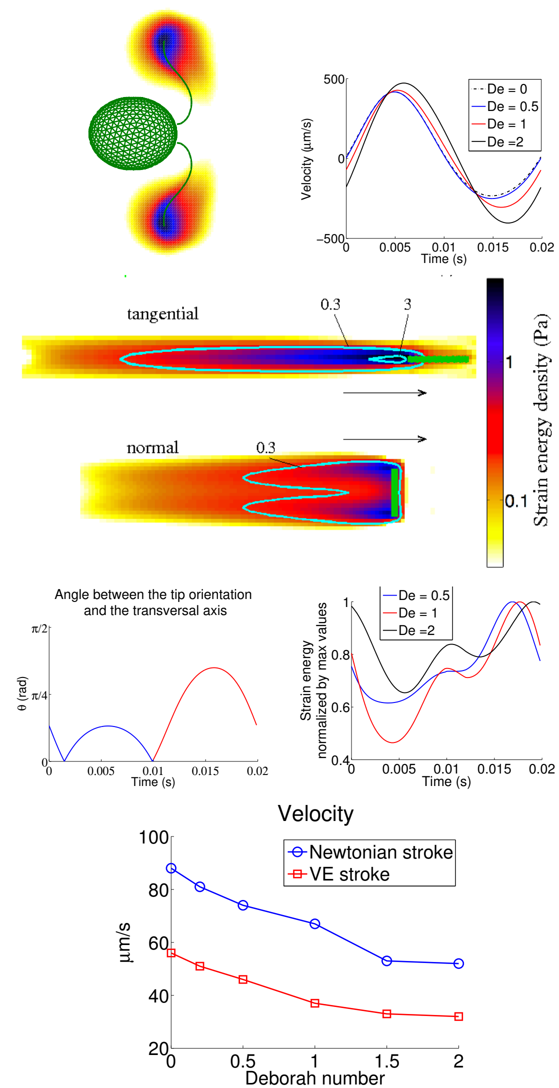




- Elastic stress from tangential motion larger than that from normal motion
- Relationship between elastic stress and orientation is reversed from Newtonian stress at high Weissenberg number

A Mechanistic Explanation of Elastic Effects on Swimming

- Large tip stresses develop on power and return stroke.
- Large elastic stress provides effective “elastic inertia”, results in enhanced both power and return stroke speeds.
- Tangential motion of cylinder results in larger elastic stress than normal motion.
- More tangential motion on return stroke, more normal motion on power stroke.
- Larger stresses, and thus more “inertia” on re-turn stroke leads to a slow-down in swimming.



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