Preliminary Exam: Calculus

University of California, Merced, January 2025

EXAM INSTRUCTIONS AND POLICIES

- You have 4 hours to answer all 8 questions on this exam.
- In order to receive full credit, you must show your work and carefully justify your answers, unless otherwise specified. Correct answers without work will receive little or no credit.
- You may use a one-sided, one page sheet of your own hand-written notes for this exam. No other notes, books, or calculators are permitted.

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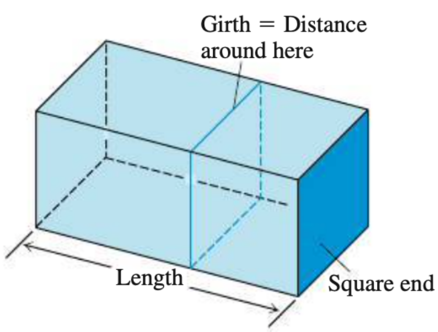
1. For the given function, find all the critical points, and classify them as local maxima, local minima, or saddle points.
$$f(x, y) = x^2 + 3y^2 - 6x - 4y$$

2. The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around), as shown in the figure, does not exceed 108 in. What dimensions will give a box with a square end the largest possible volume?



3. Evaluate **BOTH** the following integrals:

(a) $\displaystyle\int y \cdot \sqrt[3]{2y+5}\, dy$ (b) $\displaystyle\int x \ln x \, dx$

4. Evaluate **ANY ONE** of the following two limits:

(a) $\displaystyle\lim_{x \to 0^+} x \ln(x^3)$ (b) $\displaystyle\lim_{x \to \frac{\pi}{4}} \frac{\sin(4x)}{1 - \tan x}$

5. Find the volume of solid of revolution for the given region rotated about the given axis for **ANY ONE** of the following two problems:

   (a) Region bounded by the curve $y = e^x$, the line $y = e$, rotated about the x-axis.

   (b) Region bounded by the curve $y = \sqrt{x}$, the line $y = x^2$, rotated about the y-axis.

6. You only need to answer the following question for **ANY ONE** of the following series:

   Determine whether the series converges (conditionally or absolutely) or diverges. Write up your answer carefully. Clearly state the name of the convergence test strategy you use, and show that the conditions of the test are satisfied.

   (a) $\displaystyle\sum_{k=1}^{\infty}(-1)^k \frac{k^2}{k(k+50)}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^{100} \cdot 4^{3k}}{k!}$

7. Solve **ANY ONE** of the two following problems:

   (a) Let R be the region between the curves $y = x^2 - 4$ and $y = 4 - x^2$. Then

   - Sketch the region R.
   - Evaluate the integral
   $$\iint_R (x^2 + y)\, dy\, dx$$
   - Explain why the given order of integration is easier to evaluate as opposed to the order $dx\, dy$.

   (b) Consider the region bounded by the cone $z = \sqrt{3(x^2 + y^2)}$ and the hemisphere (top half of the sphere) $x^2 + y^2 + z^2 = 4$.

   - Sketch the region.
   - Set up an integral to calculate the volume of the region.
   - You *need not* evaluate the integral, but you must show all the work that leads to your integral.

8. Solve **ANY ONE** of the following two problems.

   (a) Let $\vec{F}(x, y) = \langle xe^{-2x}, x^3 + 3xy^2\rangle$. Also, let C_1 be the circle $x^2 + y^2 = 1$ and C_2 be the circle $x^2 + y^2 = 4$, both oriented counter-clockwise. Then evaluate $\displaystyle\int_{C_1} \vec{F} \cdot d\vec{r} - \int_{C_2} \vec{F} \cdot d\vec{r}$.

   (b) The plane $z = x + 4$ and the cylinder $x^2 + y^2 = 4$ intersect in a curve C. Suppose C is oriented counter-clockwise when viewed from above. Let $\vec{F}(x, y, z) = \langle x^3 + 2y, \sin y + z, x + \sin(z^2)\rangle$. Evaluate the line integral $\displaystyle\int_C \vec{F} \cdot d\vec{r}$.