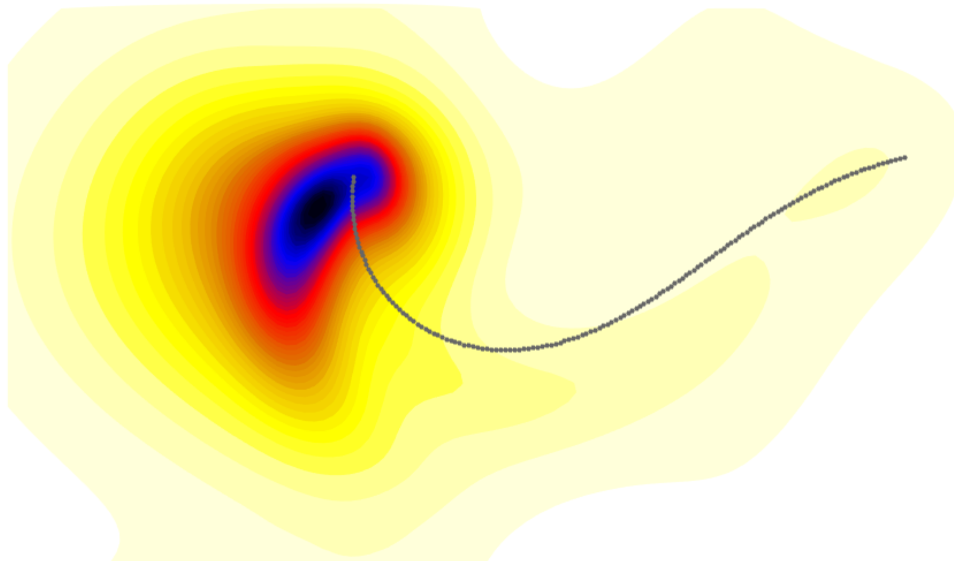
The interactions and effects of fluid and body elasticity on locomotion at the micro-scale



Yosemite Fluids Meeting 2018

SNRS Field Station

September 14-16, 2018

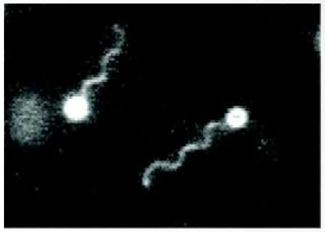
Becca Thomases

University of California, Davis

Department of Mathematics and Graduate Group in Applied Mathematics

Joint work with Robert Guy, UC Davis

Micro-organisms move in complex environments at very small scales

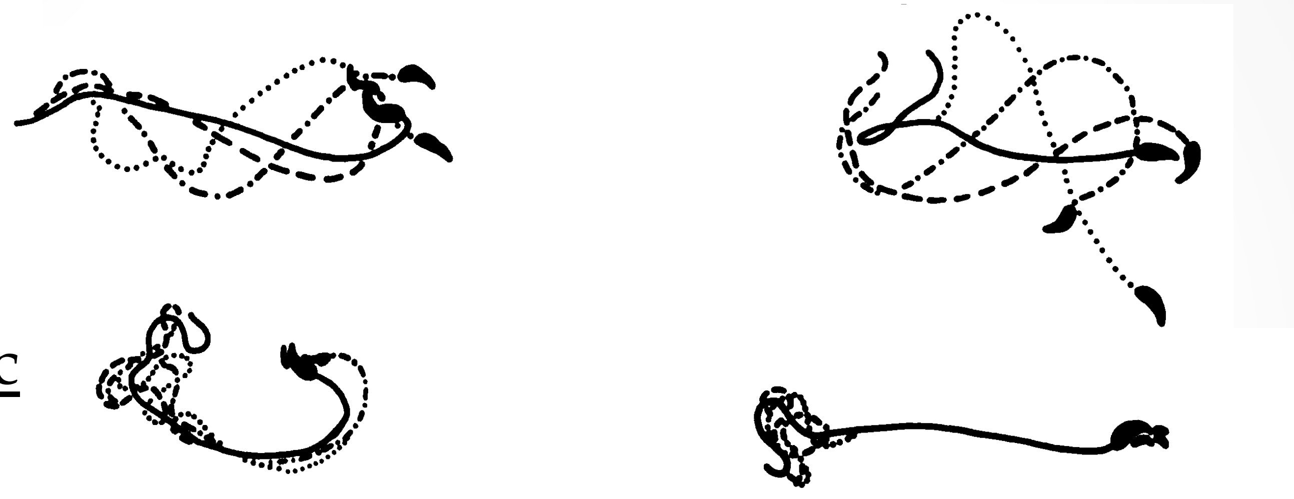
- Chlamydomonas, Sperm, C. Elegans, bacteria
- Swim in a variety of media: water, cervical mucus, soil
  - Focus on *fluids with elasticity*
- Gait changes are observed as fluid changes
  - Can be functionally important
  - Focus on *undulatory waving sheets*
- How does fluid elasticity effect swimming speed?
  - Many different results in literature
  - Incomplete picture, complicated problem
  - Nonlinear interactions of fluid, body, gait…

Motivation: Sperm Gait in Different Fluids

fresh — hyperactivated

in water

in viscoelastic fluid



From: Suarez and Dai, "Hyper-activation enhances mouse sperm capacity for penetrating viscoelastic media." *Biology of reproduction* 46.4 (1992): 686-691.

Is there a functional significance to these shapes and shape changes?

Some Results on Locomotion in Complex Fluids

Citation	Result
1979 Chaudhury (JFM) Asymptotic analysis, infinite sheet, 2nd order fluid	Speed Up
1998 Fulford, Katz, Powell (Biorheology) Resistive force theory, general linear fluid, shear thinning	Speed Up
2007 Lauga (PoF) Asymptotic analysis, infinite small amp., wavy sheet, Oldroyd-B	Slow Down
2007,2009 Fu, Powers, Wolgemuth (PRL,PoF) Asymptotic analysis, UCM/ OB, helical	Slow Down
2010 Teran, Fauci, Shelley (PRL) Simulation, finite length, undulatory, Oldoryd-B	Speed Up (Local max)
2011 Shen, Arratia (PRL) Experiment, C. elegans (undulatory)	Slow Down
2011 Liu, Powers, Breuer (PNAS) Physical Experiment , helices	Speed Up
2013 Espinosa-Garcia, Lauga, Zenit (PoF) Phyiscal experiment, flexible tail	Speed Up (monotonic)
2013 Dasgupta, Liu,Fu,Berhanu,Breuer,Powers, Kudrolli (PRE) Physical experiment, “infinite sheet” (cylinder)	Depends on rheology
2013 Spagnolie, Liu, Powers (PRL) Simulation, helices, Oldroyd-B	Speed Up
2013 Montenegro-Johnson,Smith,Loghin (PoF) Simulation, Carreau fluid	Depends on stroke
2014 Riley, Lauga: Asymptotic analysis, flexible wave sheet	Speed Up

Oldroyd-B Model for Viscoelasticity

- Stokes Equations with extra stress due to polymer stress: $\boldsymbol{\tau_p}$

$$\Delta \mathbf{u} - \nabla p + \xi \nabla \cdot \boldsymbol{\tau_p} + \mathbf{f} = 0,$$
$$\nabla \cdot \mathbf{u} = 0,$$
$$\mathrm{De}(\partial \boldsymbol{\tau_p}/\partial t + \mathbf{u} \cdot \nabla \boldsymbol{\tau_p} - \nabla \mathbf{u}\, \boldsymbol{\tau_p} - \boldsymbol{\tau_p}\, \nabla \mathbf{u}^T) + \boldsymbol{\tau_p} = \dot{\gamma}$$

ξ polymer to solvent viscosity ratio

$\dot{\gamma}$ rate-of-strain tensor

upper-convected Maxwell

- Deborah number: $De = \dfrac{\text{relaxation time}}{\text{flow time scale}}$

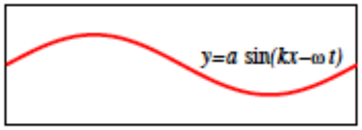
De is a measure of elasticity of the fluid

$De \to 0$ recover Newtonian fluid

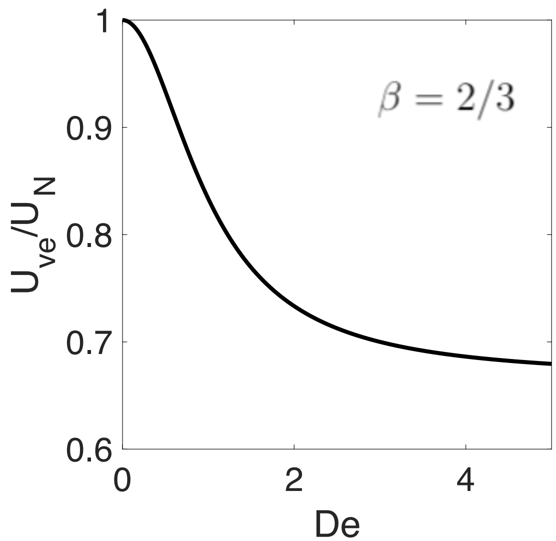
$De \to \infty$ recover neo-Hookean elastic solid

- Oldroyd-B model can also be derived from dilute suspension of dumbbells connected by linear springs

Undulatory Swimmers in viscoelastic fluid

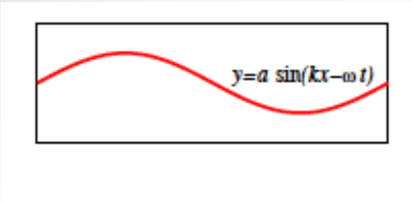


1. Asymptotic analysis: infinite length, small amplitude
   - 2007, Lauga: Always slow down



$$\frac{U_{\mathrm{ve}}}{U_N} = \frac{1+\beta \mathrm{De}^2}{1+\mathrm{De}^2}, \quad \beta = \frac{\eta_s}{\eta_p + \eta_s} < 1$$

Undulatory Swimmers in viscoelastic fluid



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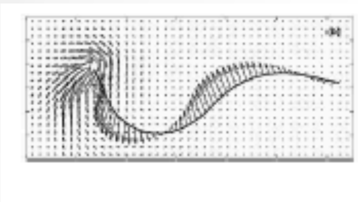
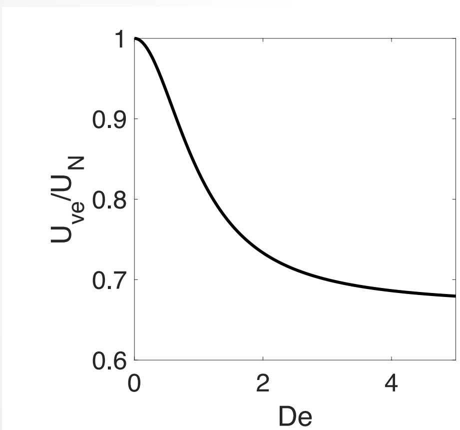
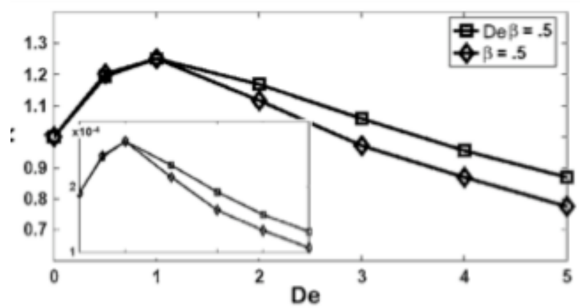
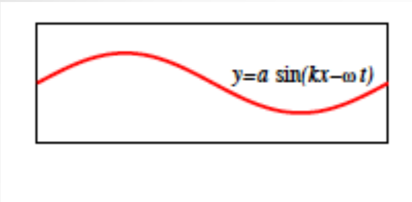


2. Simulations: finite length, large amplitude
   - 2011, Teran-Fauci-Shelley: Non-monotonic speed up





Undulatory Swimmers in viscoelastic fluid



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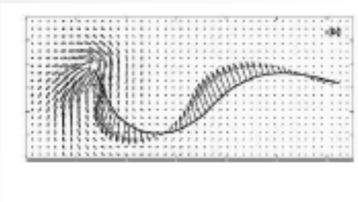

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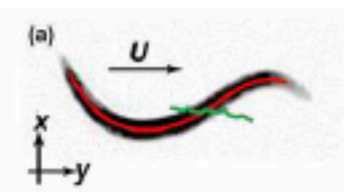

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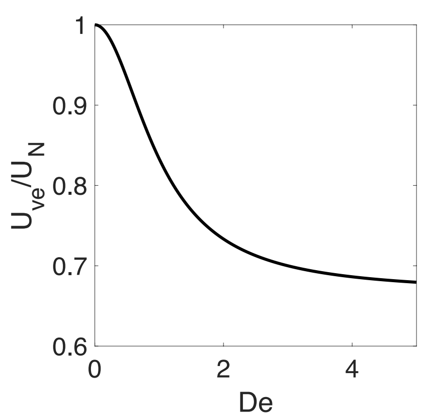
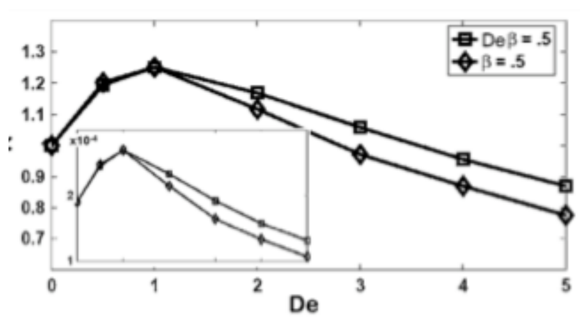
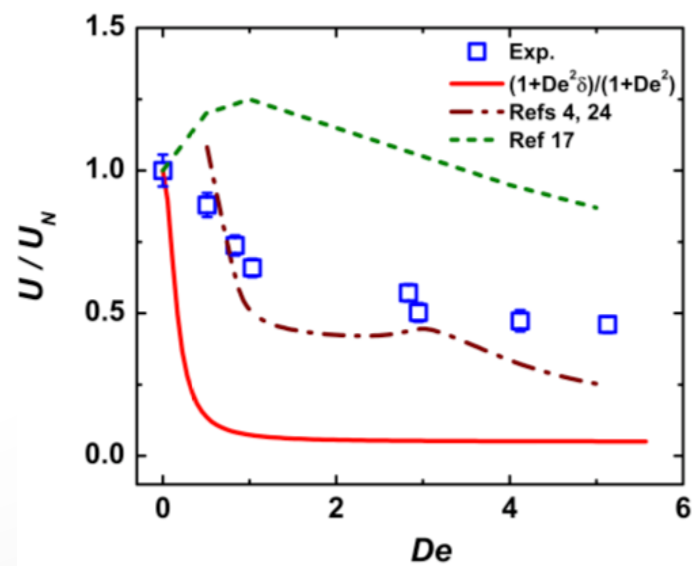
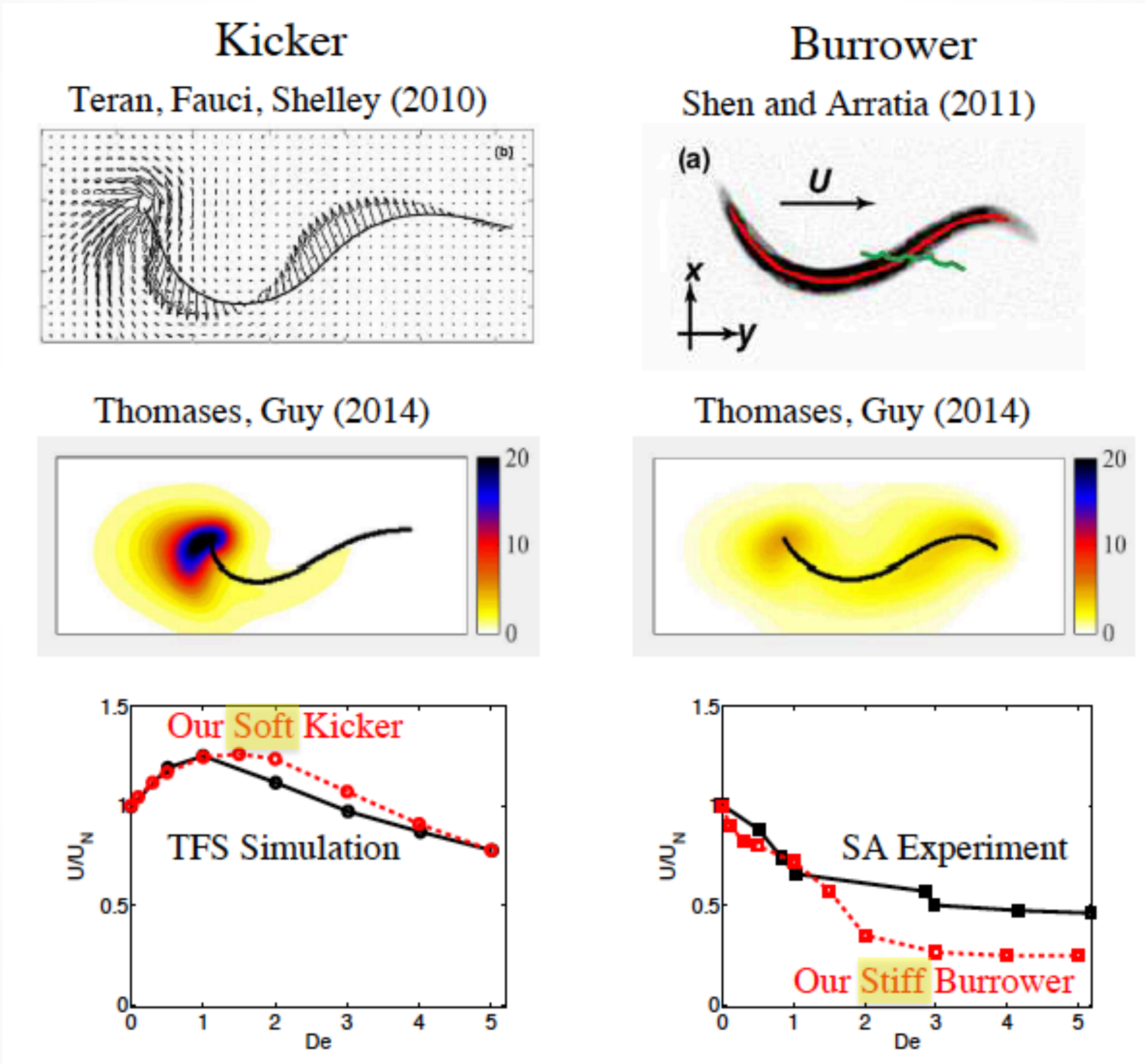






Undulatory Swimmers in viscoelastic fluid



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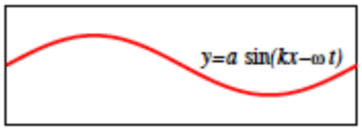

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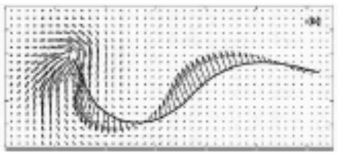

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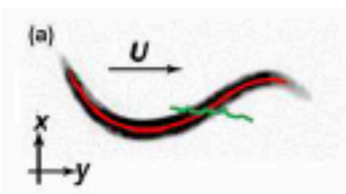
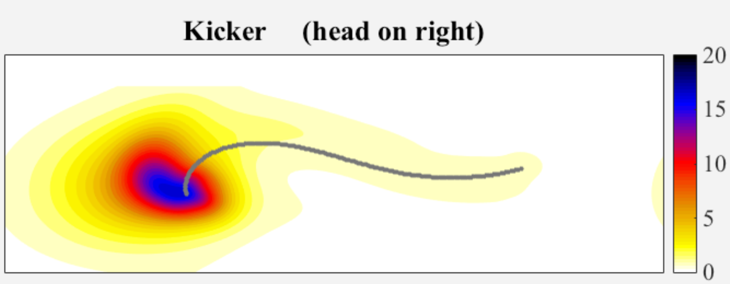
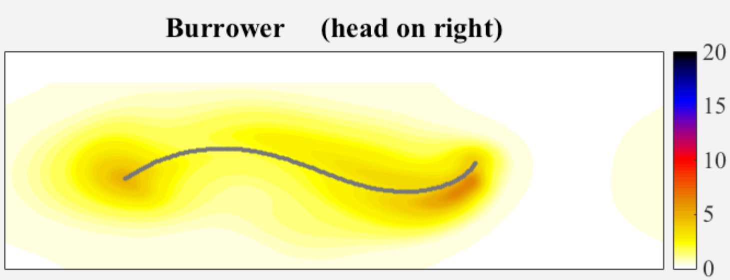


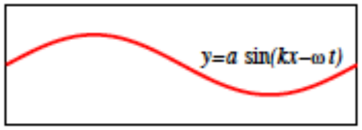
3. Biological experiment (C. Elegans): finite length, large amplitude
   - 2011, Shen-Arratia: Always slow down

2014, Thomases-Guy: The gait is really important!





Undulatory Swimmers in viscoelastic fluid



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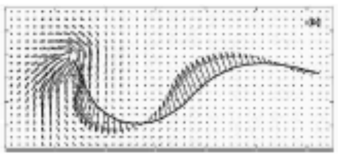

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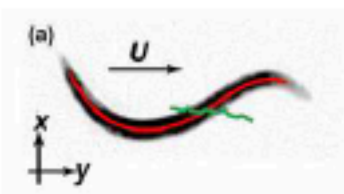

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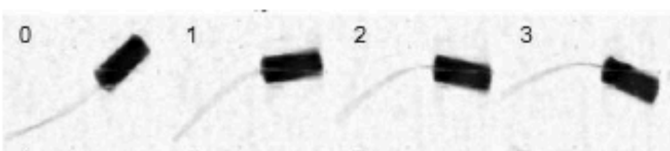
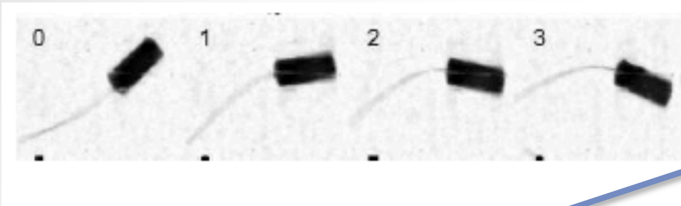


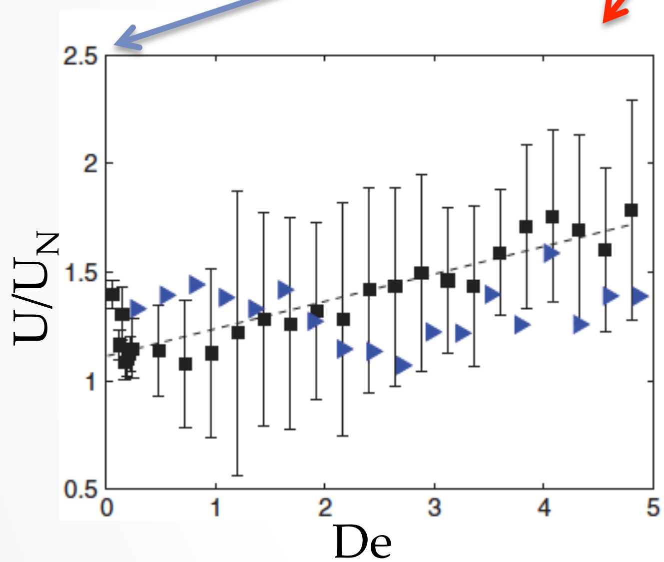
4. Physical experiment: finite length, large amplitude
   - 2013, Espinosa-Garcia, Lauga, Zenit: Monontonic speed up

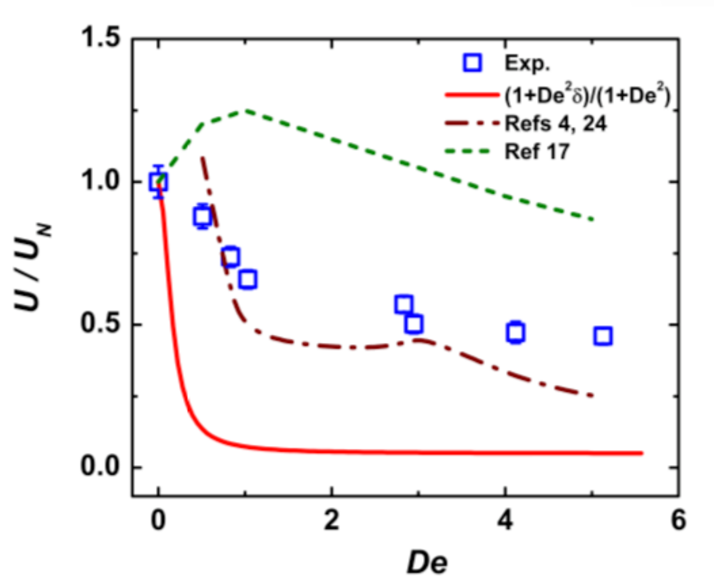
Undulatory Swimmers in viscoelastic fluid



4. Physical experiment: finite length, large amplitude
    - 2013, Espinosa-Garcia, Lauga, Zenit:

BIG Monontonic speed up





Model Equations

- Fluid-Equations – Stokes-Oldroyd-B model

$$\Delta \mathbf{u} - \nabla p + \xi \nabla \cdot \boldsymbol{\tau_p} + \mathbf{f} = 0,$$
$$\nabla \cdot \mathbf{u} = 0,$$
$$\mathrm{De}(\partial \boldsymbol{\tau_p}/\partial t + \mathbf{u} \cdot \nabla \boldsymbol{\tau_p} - \nabla \mathbf{u}\, \boldsymbol{\tau_p} - \boldsymbol{\tau_p}\, \nabla \mathbf{u}^T) + \boldsymbol{\tau_p} = \dot{\boldsymbol{\gamma}}$$

- Structure Equations – penalty method (Immersed Boundary)

$$X_t = \int_{\text{fluid}} \delta(x - X(s,t)) u(x,t)\, dx \qquad f_{\text{struct}} = \int_{\text{body}} \delta(x - X(s,t)) F(s,t)\, ds$$

$$F = \frac{\delta E}{\delta X} \qquad E = E_b + E_s$$

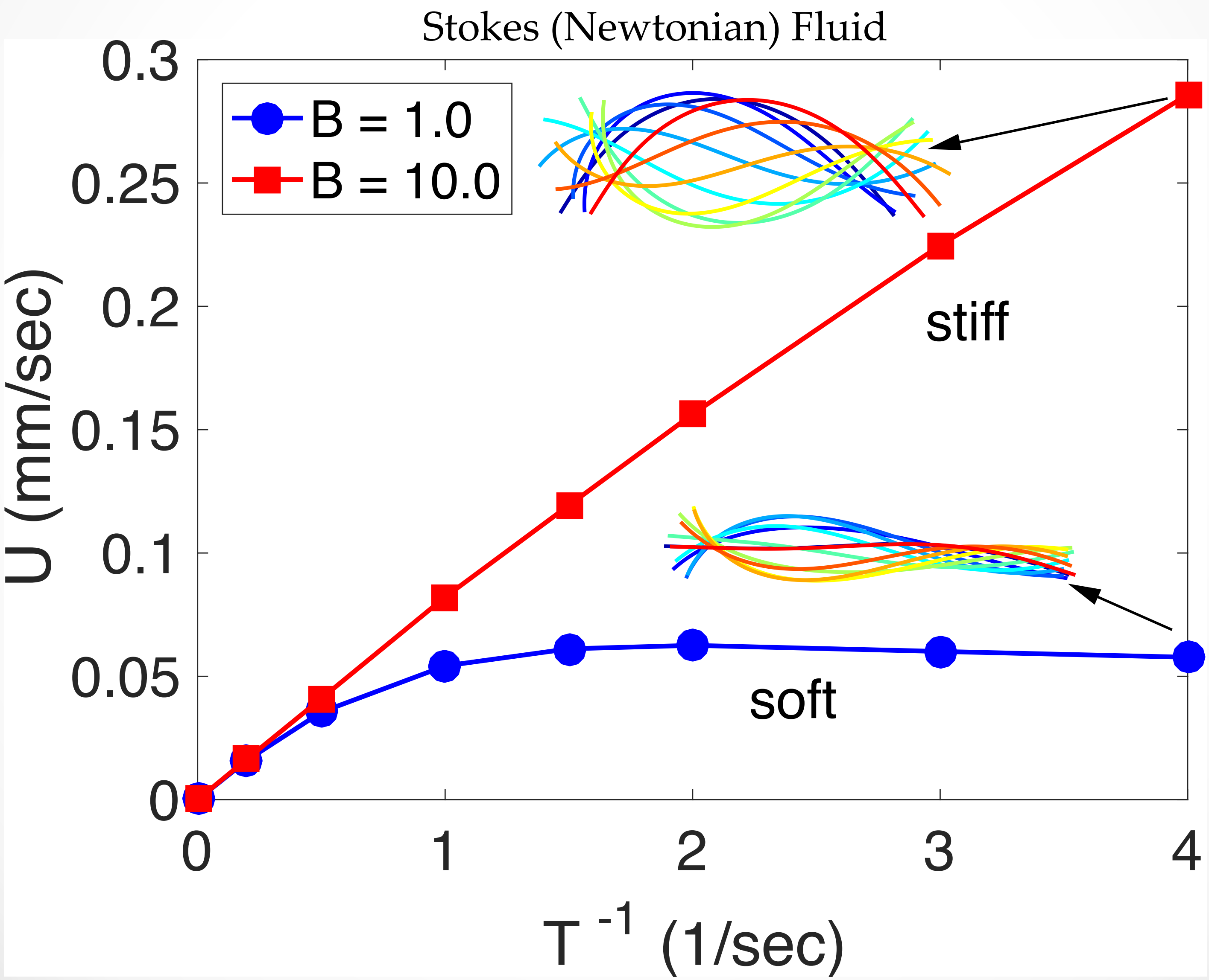
E_s, stiff penalty for stretching "easy" to enforce

$$\boxed{E_b = \frac{B}{2} \int_{\text{worm}} (\kappa - \kappa_0(s,t))^2\, ds}$$

Prescribed body moments along the swimmer

- Rigid body: $B \gg 1$ - realized shape is very close to prescribed shape
- Flexible body: $B \approx 1$ - elastic forces and viscous forces are of the same scale and resultant shape is result of fluid-structure interaction

Speed dependence on frequency is related to swimmer body stiffness

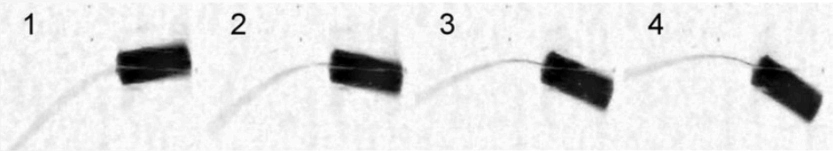


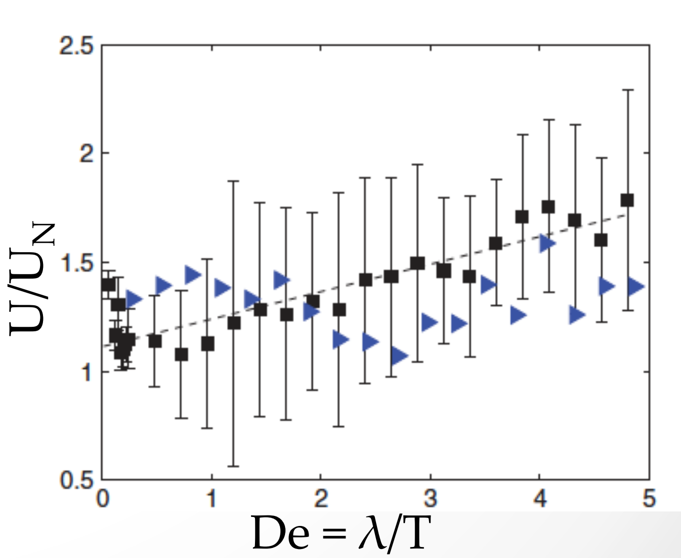
Fluid elasticity measured with Deborah number

Change the Deborah number by changing the fluid **or** changing the period of oscillation

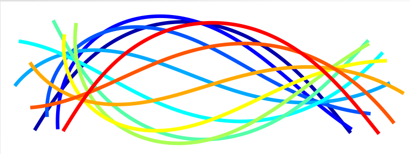
$$De = \frac{\lambda}{T} = \frac{\text{Viscoelastic relaxation time}}{\text{Period of oscillation}}$$

- Numerical Simulations: Scale time by the period and thus change the relaxation time (Teran-Fauci-Shelley, Thomases-Guy)
- Biological experiment: (Shen-Arratia) Cannot control period and must change fluid (relaxation time)
- Physical model: E-L-Z (2013)
- **Changed the period**



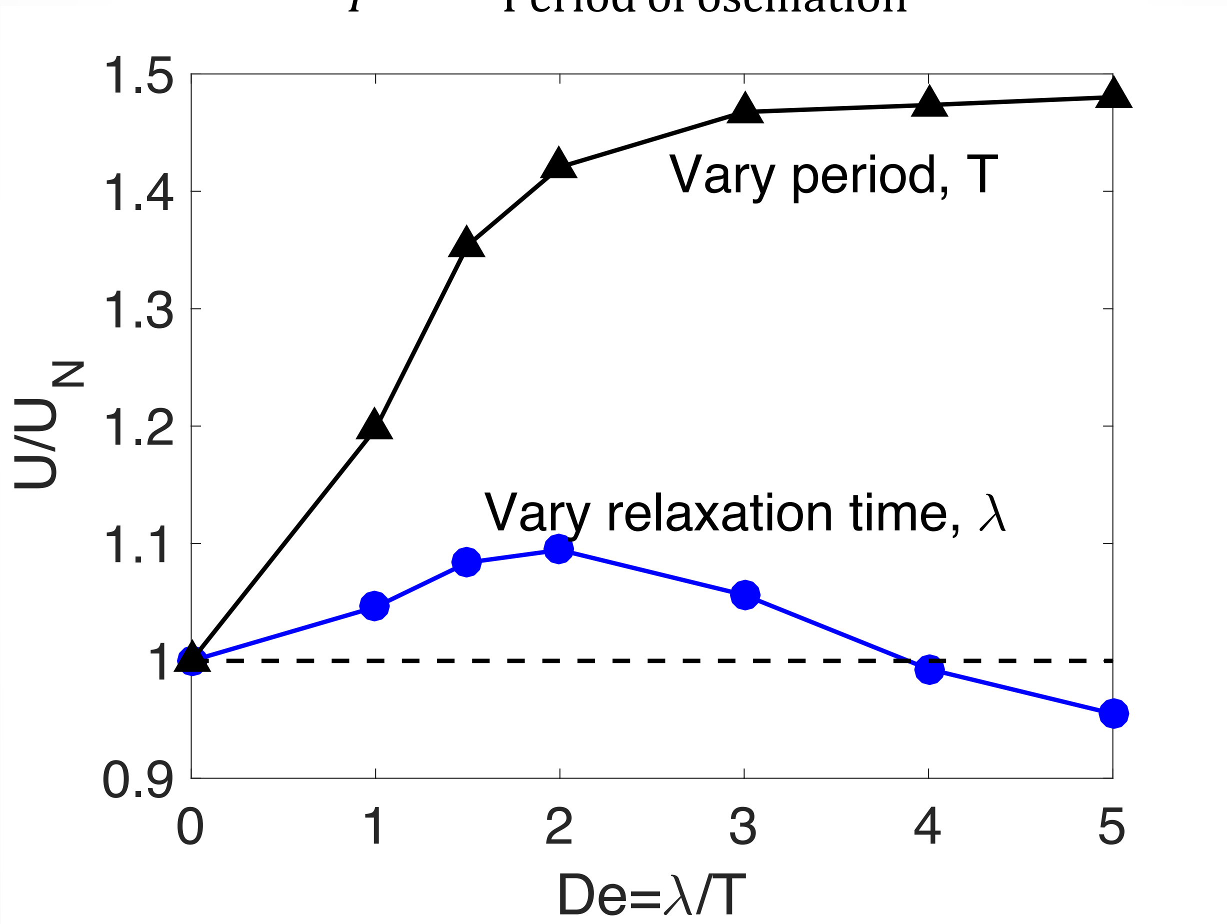


Swimming speed for *soft kicker*



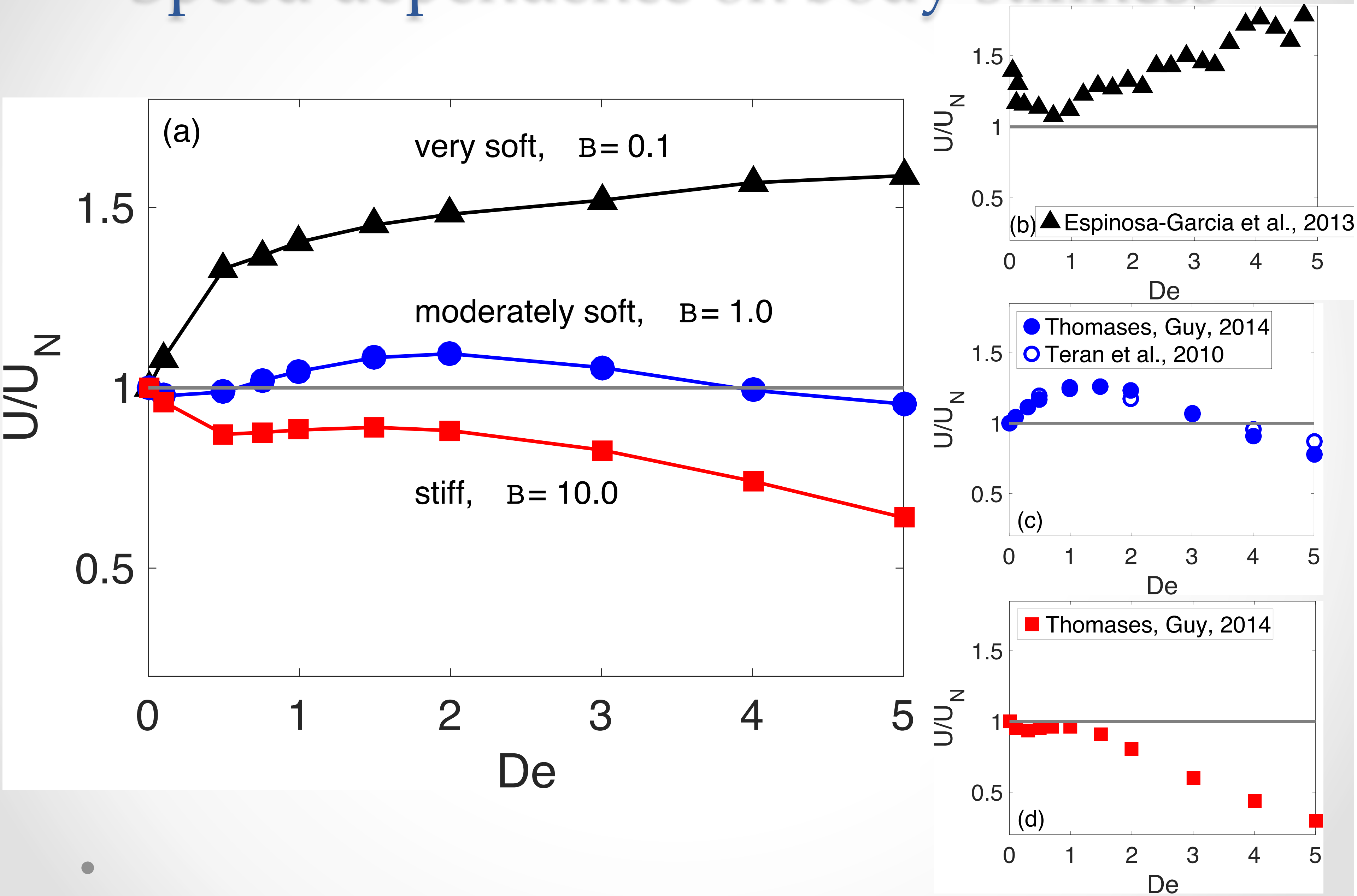
aka large amplitude tail, flexible body (B~1)
vary De "two ways"

$$De = \frac{\lambda}{T} = \frac{\text{Viscoelastic relaxation time}}{\text{Period of oscillation}}$$



Deborah number alone is not the whole story!

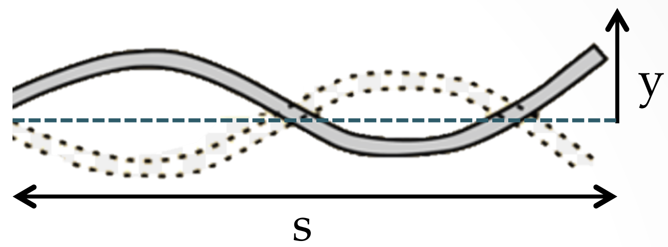
Speed dependence on body stiffness



Elasto-hydrodynamics

- Balance of viscous drag force with elastic rod force

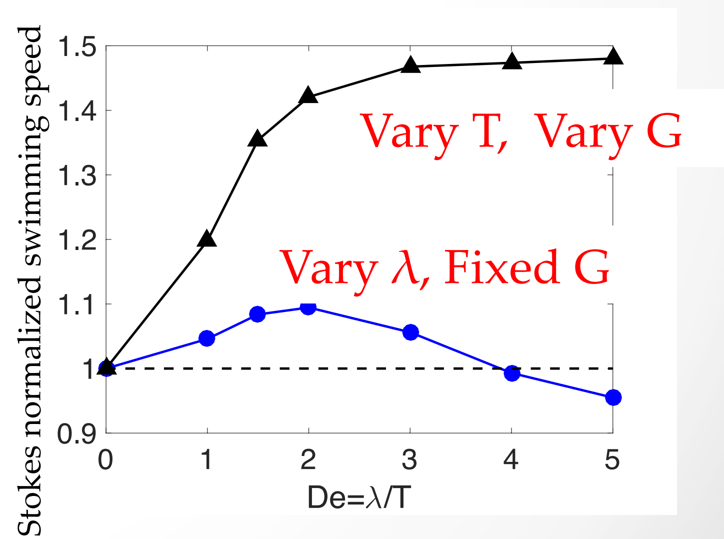
$$\boxed{\zeta \frac{\partial y}{\partial t} = -B \frac{\partial^4 y}{\partial s^4}}$$



- Define a dimensionless (***inverse) body relaxation time, G***:

$$G = \frac{T}{B^{-1}\zeta L^4} = \frac{\text{Period of oscillation}}{\text{beam memory time}}$$

$$(G = Sp^{-4}, \quad \text{sperm number})$$



Elastic induced shape changes

- Elastic induced shape changes depend on ***G*** and ***De***
- Small amplitude theory: shape changes first order in amplitude, swimming speed is second order
- Use linear viscoelasticity
  - Fulford et. al (1998), **Fu-Wolgemuth-Powers (2007,2008)**
- Obtain a *complex drag coefficient,* depends on fluid

$$\zeta_{viscoelastic} = \left(\frac{1+\eta_p / \eta_s + 2\pi i \mathrm{De}}{1+2\pi i \mathrm{De}} \right) \zeta_{viscous}$$

- Solve for shape changes as a function of driving curvature
- Can we do analysis for LARGE AMPLITUDE?

Large Amplitude: Motion of curvature deviations

- Drive our system with prescribed target curvature
- Derive PDE for ***curvature deviations*** (good approximation for small amplitude or high stiffness limit)

$$c_t \approx -Gc_{ssss} - \frac{\partial \kappa_0}{\partial t}$$

- Solve for shape as a function of prescribed curvature

$$\alpha_k = \alpha_k^{\infty}\left(1-\left(1-\frac{G\mu_k}{\zeta_{ve}2\pi i}\right)^{-1}\right)$$

resulting shape (amplitude)

prescribed shape (amplitude)

$$\zeta_{ve} = \left(\frac{1+\eta_p/\eta_s+2\pi i \mathrm{De}}{1+2\pi i \mathrm{De}}\right)\zeta_{vis}$$

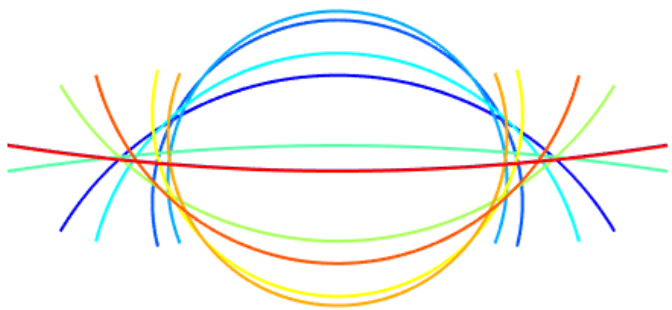
Flexors: non-translating target curvature

Consider a flexible beam that does not translate horizontally, prescribe target curvature:

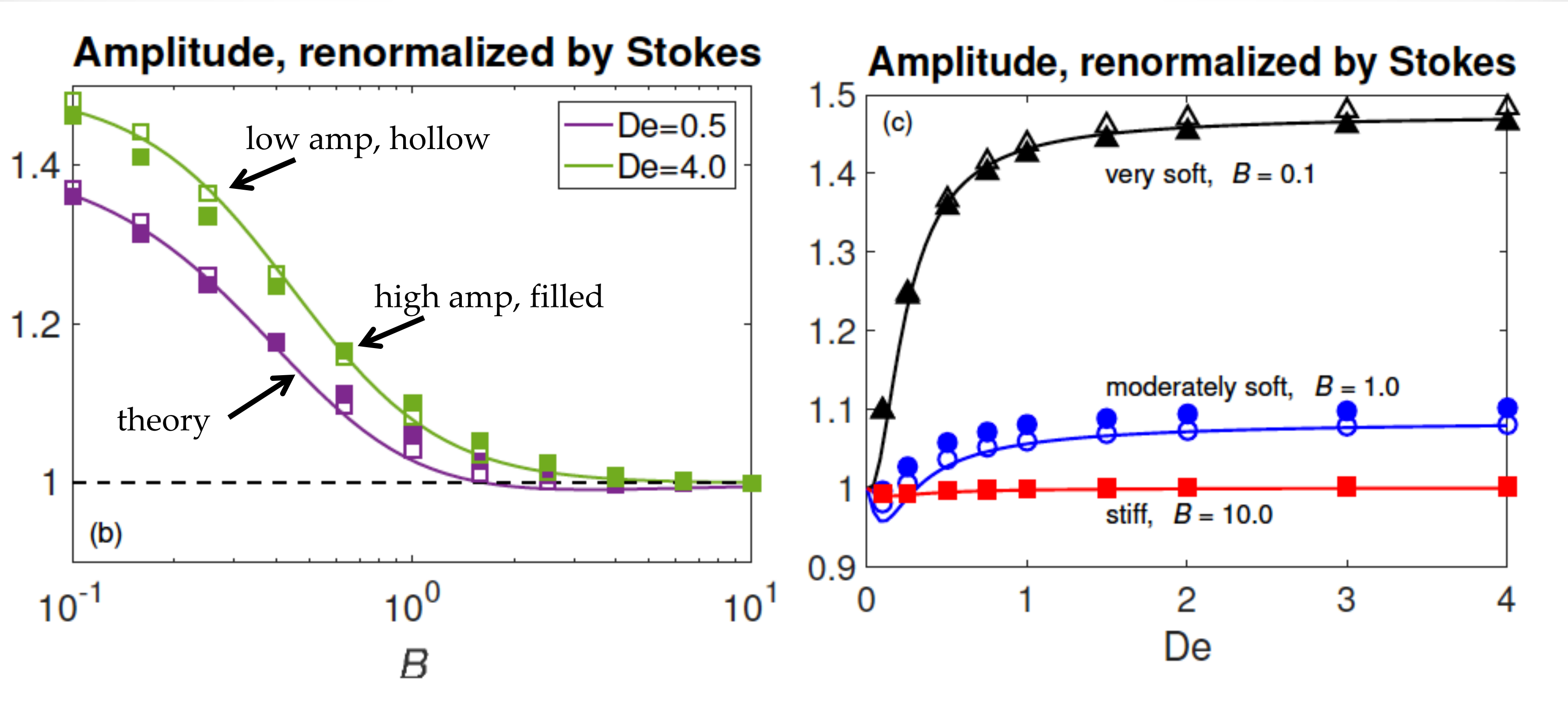
$$\kappa_0(s,t) = A\sin(\omega t)$$

A=0.5

A=4.0

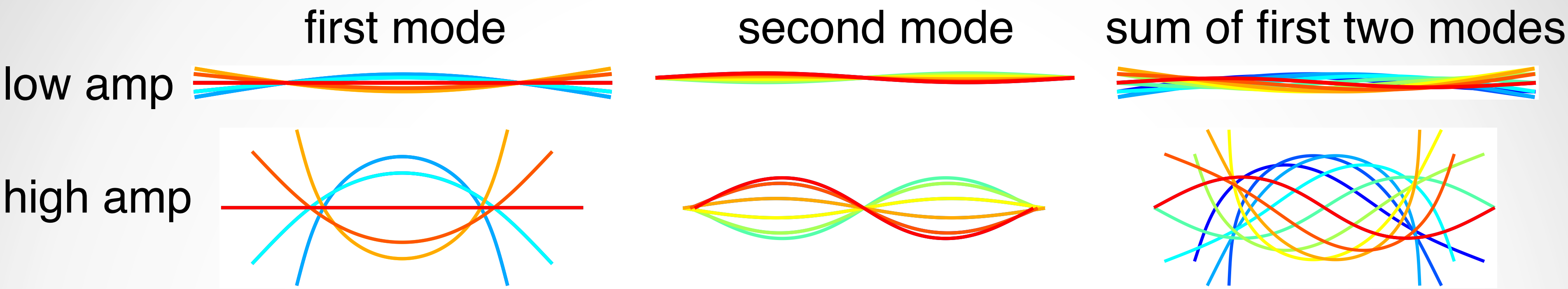


Flexors: compare simulation and theory



Emergence of three regimes, a very soft regime where the amplitude is always boosted, a moderately soft regime with a non-monotonic response, and a stiff regime with no amplitude boost

What about swimming?



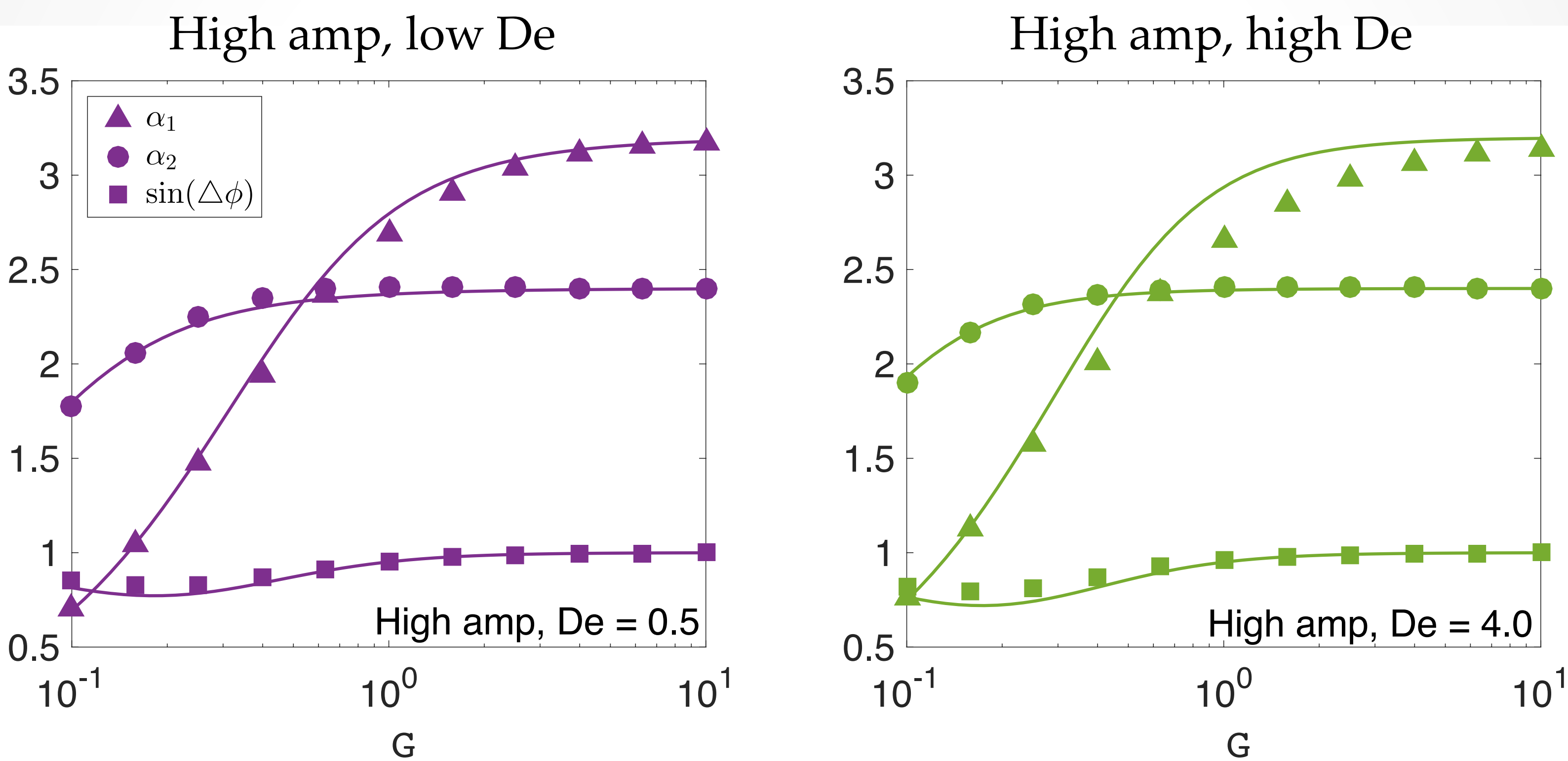
$$\kappa_0(s,t) = \alpha_1 \cos(2\pi t / T + \phi_1)\psi_1(s) + \alpha_2 \cos(2\pi t / T + \phi_2)\psi_2(s)$$

Theoretical swimming speed **in viscous fluid** can be computed:

$$\langle U \rangle \propto \left(\frac{\zeta_\perp}{\zeta_{||}} - 1 \right) \frac{1}{LT} \int_0^T \int_0^L y_x y_t \, ds \, dt$$

$$\langle U \rangle \propto \alpha_1 \alpha_2 \sin(\phi_2 - \phi_1)$$

Shape Comparisons: simulation and theory (Low amplitude is spot on)

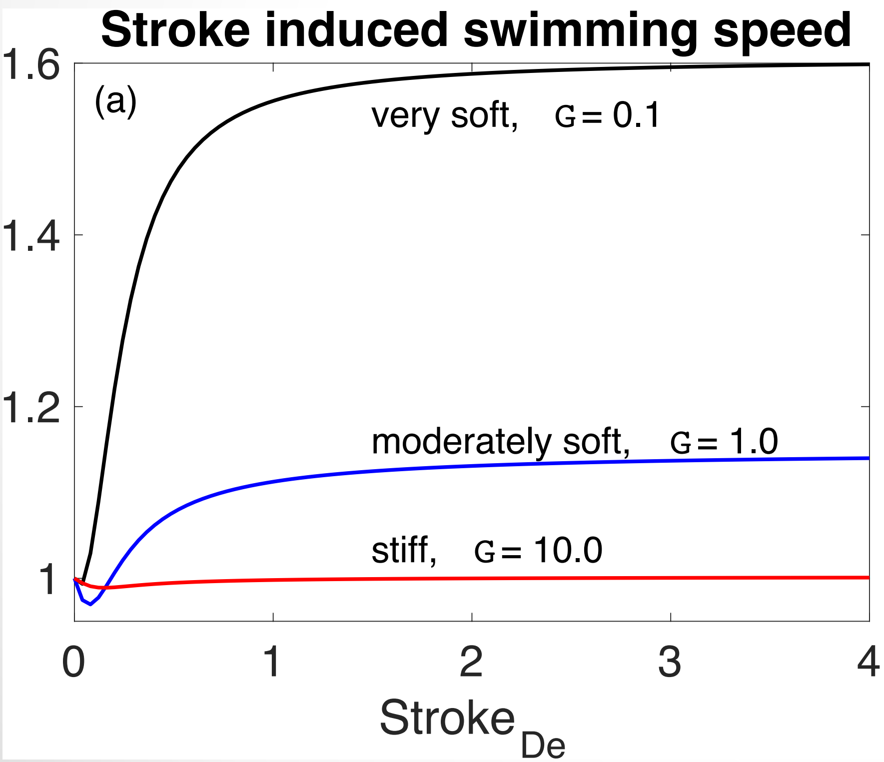


Like the flexor the theory predicts the shape changes very well!

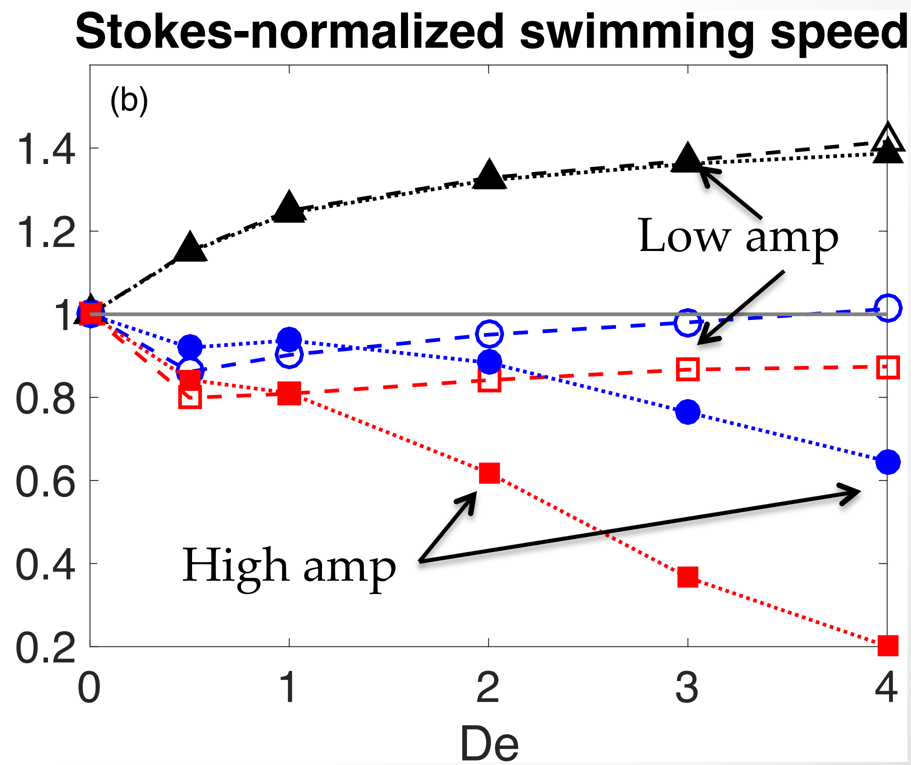
Theoretical vs. simulation swimming speed

Theory – Speed changes due to shape changes

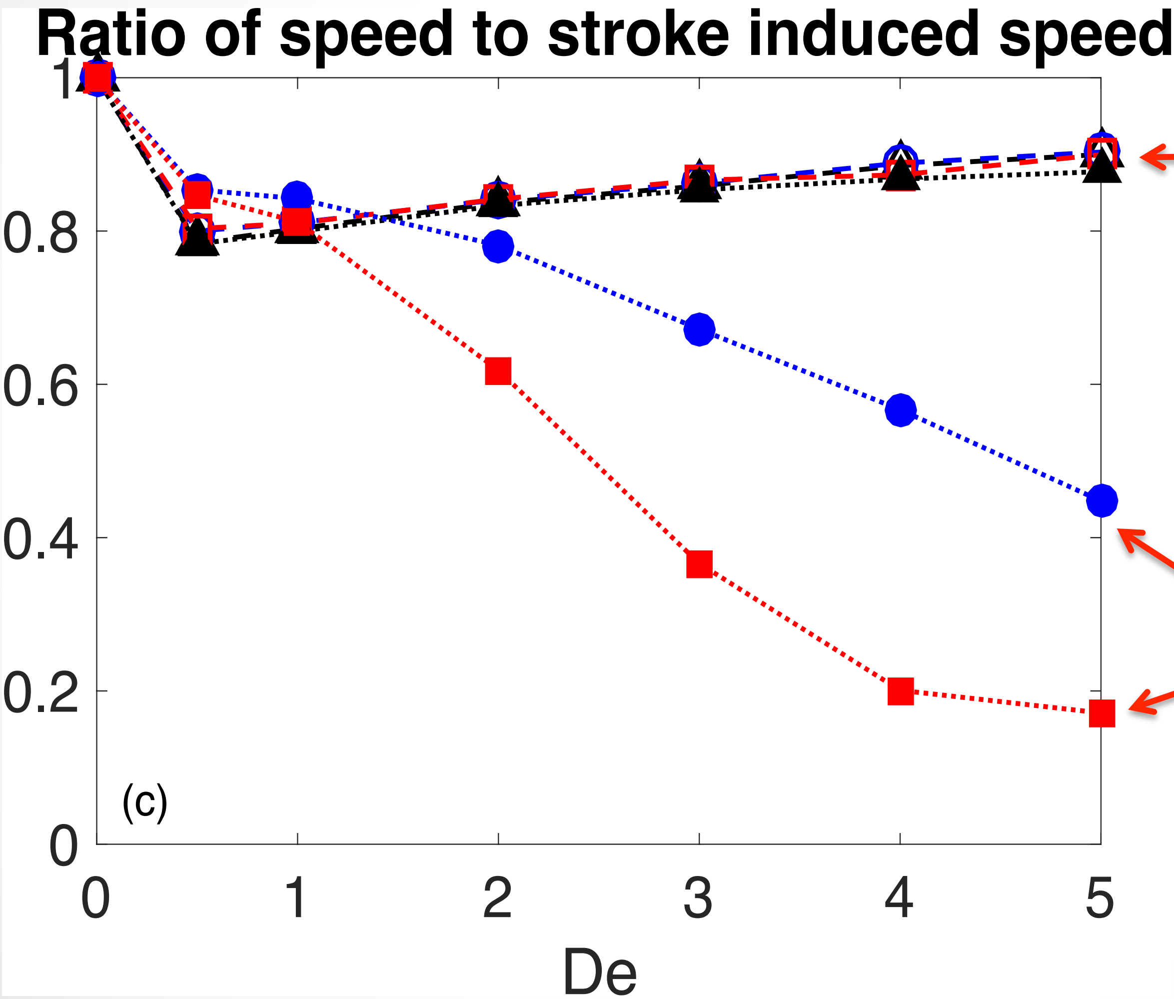
$$\propto \alpha_1 \alpha_2 \sin(\phi_2 - \phi_1)$$



Simulations include other fluid effects



What is the "fluid" effect?
i.e. minus shape changes



Data collapses for low amplitude

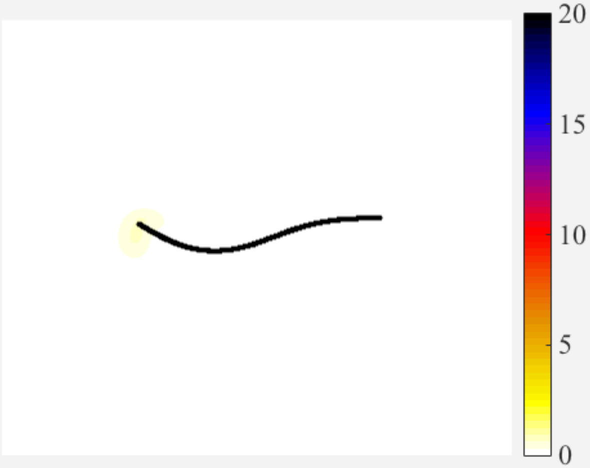
This effect is likely to be sensitive to specifics of the stroke

Something else is going on in the high amplitude case

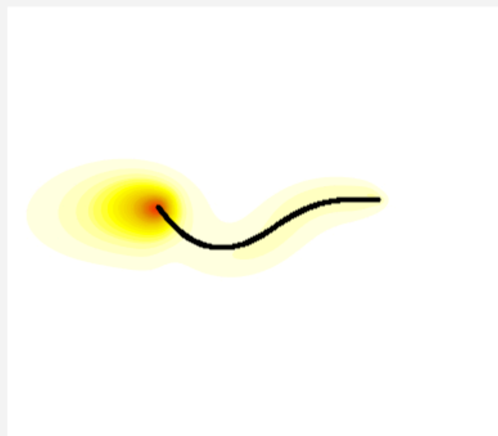
Theory ignores nonlinear elasticity effects

De = 4.0

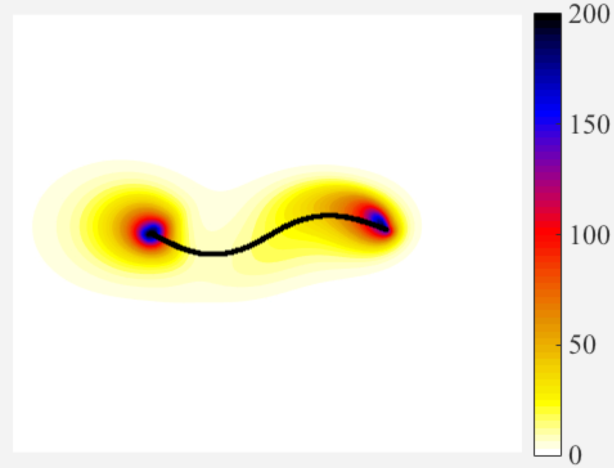
Softest, G=0.1



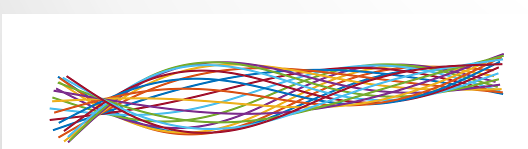
Moderate, G=1.0

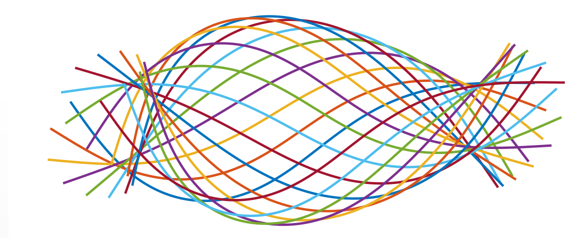


Stiffest, G=10.0

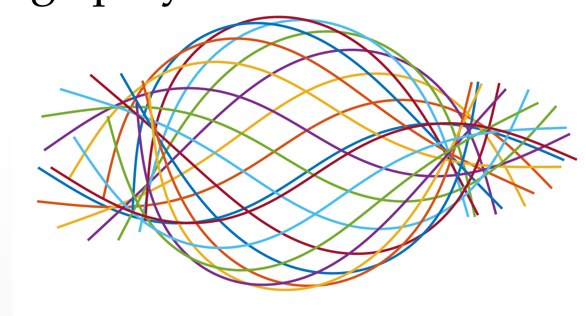


Target stroke not achieved.
Effectively low amplitude
and low polymer stress.

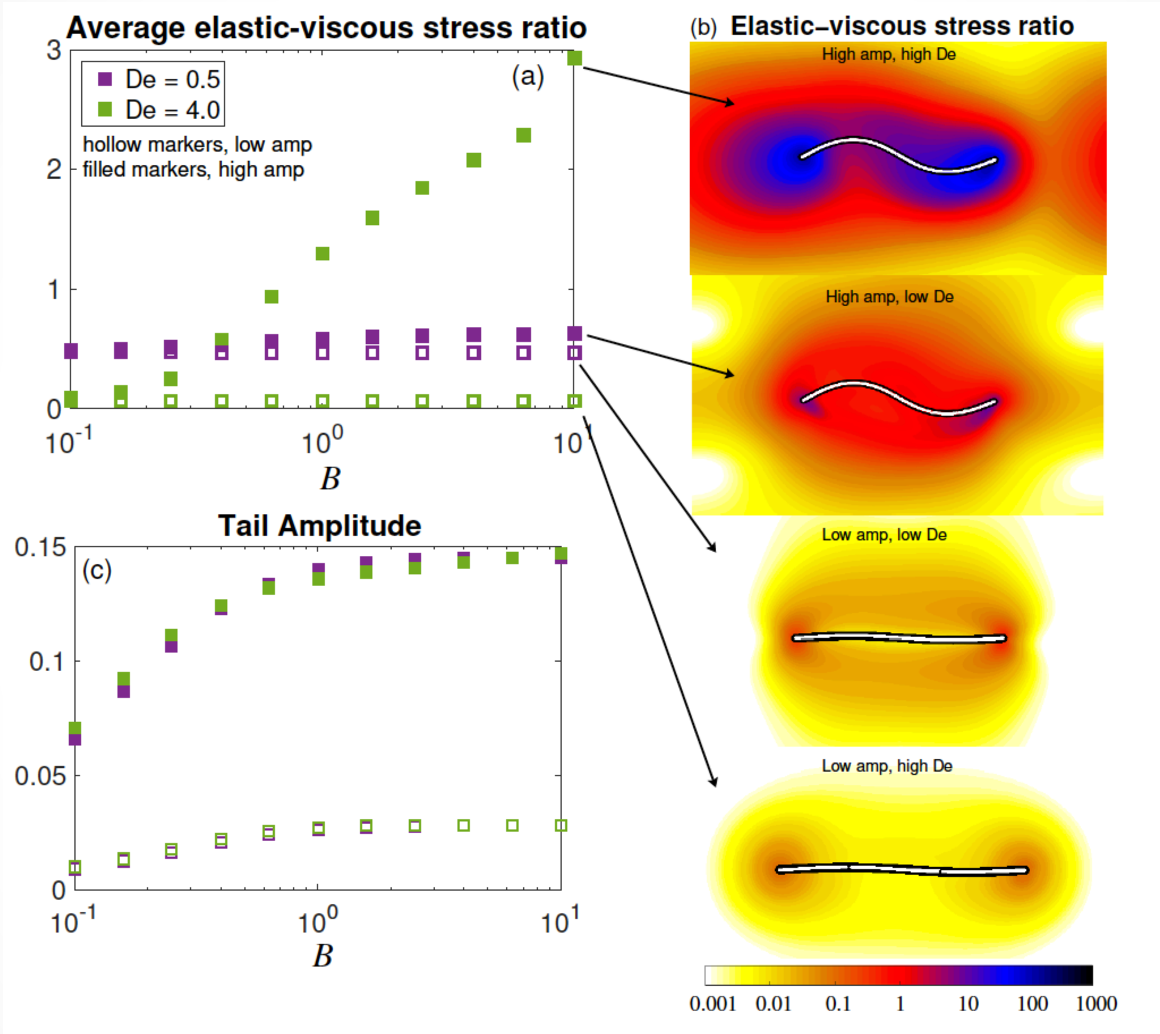




Target stroke strongly enforced.
Large polymer stress as a result.



Look at a measure of amplitude along side stress ratios



It is only in the high amplitude and high De regime where very large stresses develop

Some conclusions

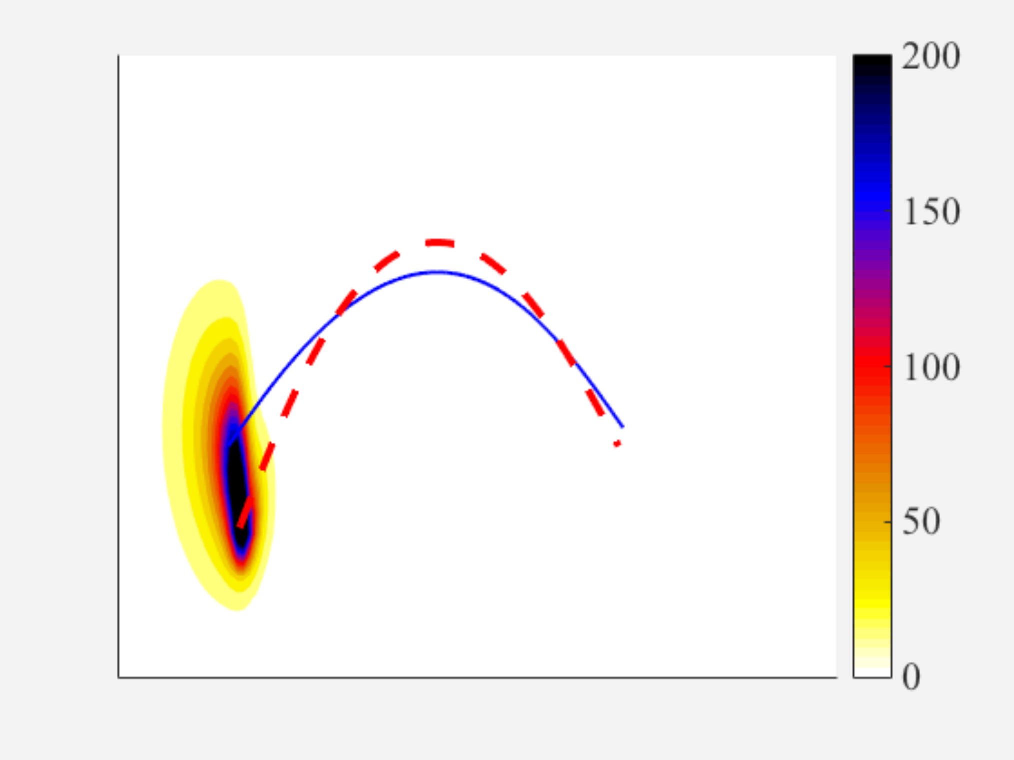
- Espinosa-Garcia, Lauga, Zenit result has $G \approx 0.43$, soft regime, expect speed ups

- Another numerical group reported only slow-downs,

  2016, Salazar, Roma, Ceniceros, $G \approx 7.7B$, stiff regime ($L = 0.6$, $G \sim B/L^4$), expect slow downs

- Essential to report body relaxation time:
  - 2007 Lauga – small amplitude, large G, always slow down
  - 2014 Riley, Lauga – small amplitude, vary G, speed up possible
  - 2010 Teran, Fauci, Shelley – large amplitude, small G, sometimes speed up
  - 2011 Shen, Arratia- large amplitude (head), large G, always slow down
  - 2013 Espinosa-Garcia, Lauga, Zenit – large amplitude, small G, always speed up
  - 2016 Salazar, Roma, Ceniceros, large G, always slow down

Some questions



- Away from the tail: linear elastic fluid assumption valid?
- What causes the large tail stresses?
- What is the effect on swimming from the large tail (or head) stresses?