Directions: You have 4 hours to complete this exam. Show all of your work on separate pieces of pear and circle your final answers where appropriate. When you are finished, staple your work to this sheet and put your name on it. You are allowed one hand-written crib sheet but NO calculators, phones, or other study aides allowed. Remember to explain your work clearly and legibly so that you may receive full credit. Good luck!!!

1. (*20 points*) For the following limits, either calculate the correct value or explain why the limit does not exist:

$$a)\ \lim_{x\to\infty}\left(1+\frac{2}{3x}\right)^{3x} \qquad b)\ \lim_{x\to-\infty}\frac{\sqrt{2x^2-2}}{x+5} \qquad c)\ \lim_{x\to-1}\frac{x+1}{|x+1|} \qquad d)\ \lim_{x\to0}x^2\cos\frac{1}{x}$$

2. (*15 points*) Compute the following integrals:

$$a)\ \int_0^2\frac{x}{1+x^4}\,dx \qquad b)\ \int e^x\sin x\,dx \qquad \int_{-\pi}^{0}\sqrt{1-\cos^2 x}\,dx$$

3. (*10 points*) Does the series $\sum_{n=1}^{\infty} ne^{-\sqrt{n}}$ converge or diverge? Justify your answer.

4. (*10 points*) Find the Maclaurin series for the function $\int_0^x e^{-t^2}\,dt$.

5. (*10 points*) Find a series representation of $\ln x$ in powers of $x-1$.

6. (*10 points*) The temperature distribution on the surface $x^2+y^2+z^2=1$ is given by $T(x,y,z)=xz+yz$. Find the hottest spot.

7. (*10 points*) Find the work done by the force $\mathbf{F}(x,y)=(1+\tan x)\mathbf{i}+(x^2+e^y)\mathbf{j}$ on the curve $\mathcal{C}$ where $\mathcal{C}$ is the boundary of the region lying between the graphs of $y=\sqrt{x}, y=0, x=1$, oriented counterclockwise.

8. (*10 points*) Find the *outward flux* Φ of the vector field $\mathbf{F}(x,y,z)=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ through the surface of the cone $z=\sqrt{x^2+y^2}$, $1\le z\le 2$.

9. (*10 points*) Find the circulation of the vector field $\mathbf{F}(x,y,z)=3xz\mathbf{i}+(7x+6yz)\mathbf{j}+3x^2\mathbf{k}$ around the path $\mathcal{C}$ which is the circle $x^2+y^2=4, z=4$ oriented counterclockwise when viewed from above.

10. (*10 points*) You have been asked to find the path along which a force field $\mathbf{F}$ will perform the least work in moving a particle between two locations. A quick calculation on your part shows $\mathbf{F}$ to be conservative. How should you respond? Give reasons for your answer.