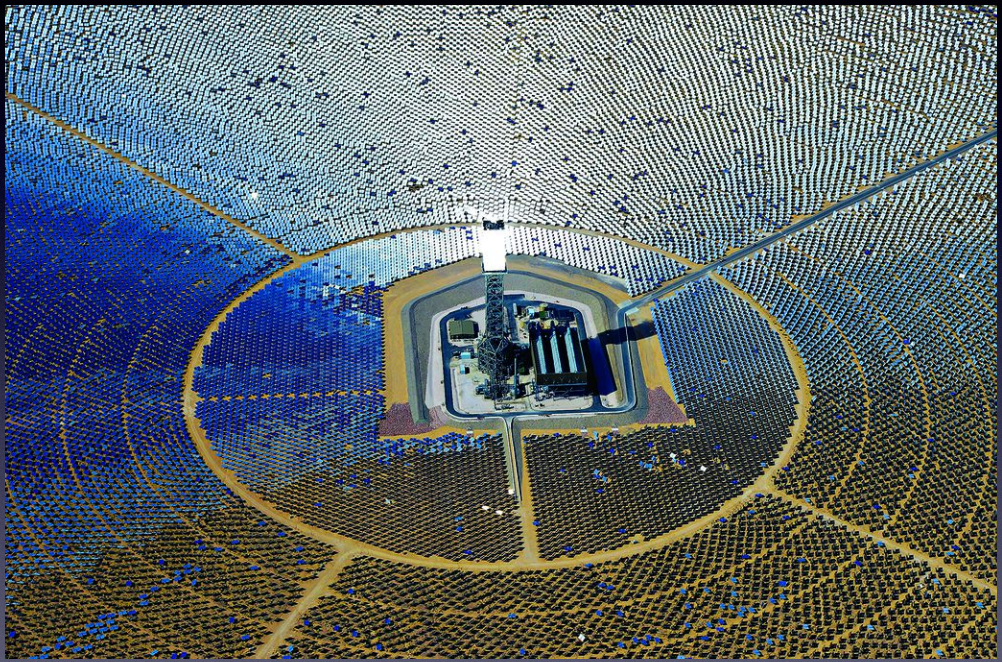
Green’s Function for Nonimaging Optics

Boaz Ilan

Ivanpah Solar Power Facility



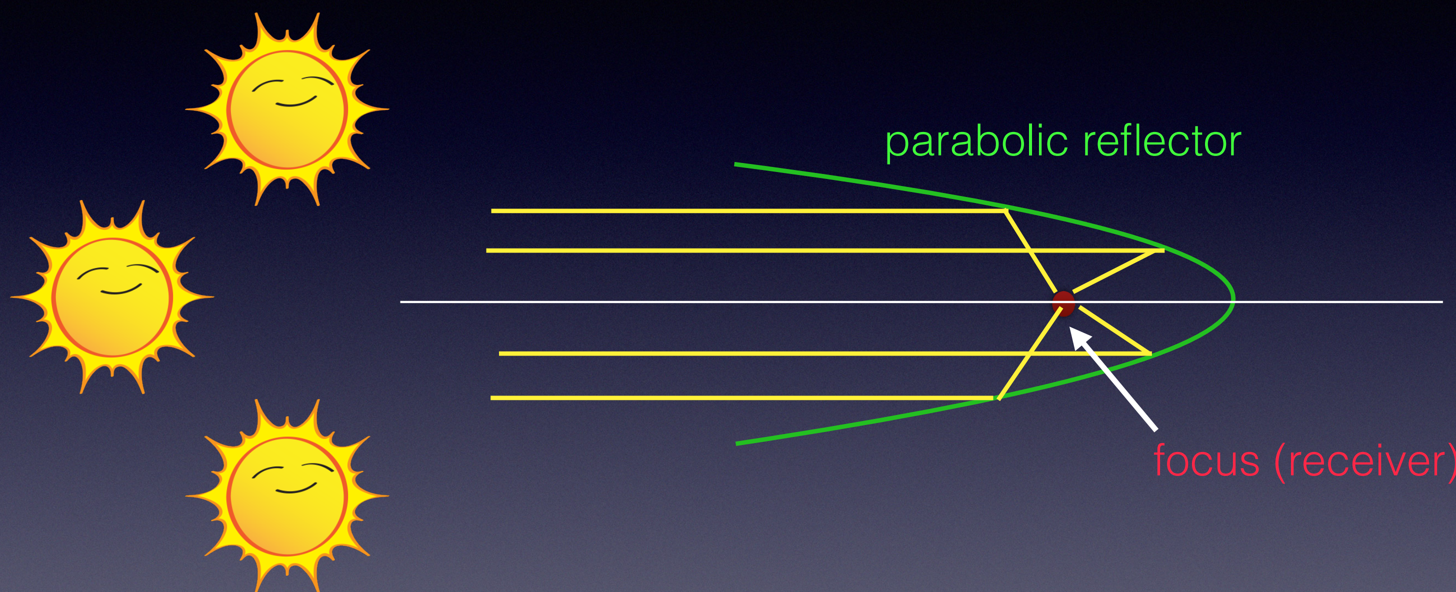
- 173,500 heliostats
- 140 meter tower $2.2 billion
- 392 MW

A Huge Solar Plant Caught on Fire, and That's the Least of Its Problems

SAN BERNARDINO COUNTY FIRE DEPARTMENT

*Update: On Wednesday, May 25, Ivanpah's operator, NRG energy, confirmed the fire was indeed caused by mirrors that did not track the sun properly, which focused sunlight onto the wrong part of the tower. NRG spokesperson David Knox estimates the plant would be back online within three weeks. *

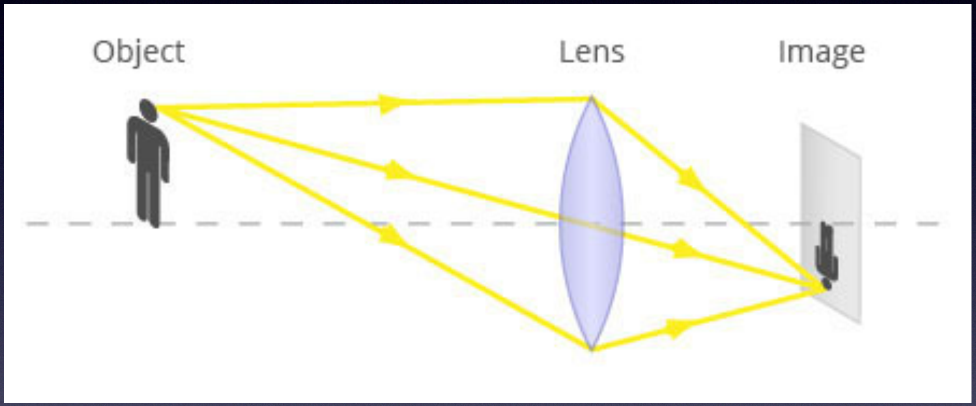
Concentrated Solar



Limitations:

- Need to track the sun
- Do not collect diffuse light

Imaging Optics



What is an image?

Imaging Optics

Maxwell's equations for electromagnetic waves

- Monochromatic waves
- One polarization component
- High-frequency

Get Eikonal equation

$$|\overrightarrow{\nabla} S(\mathbf{x})| = n(\mathbf{x})$$

- S is the phase of the wave
- n is refractive index

Ray Optics

$$|\vec{\nabla} S(\mathbf{x})| = n(\mathbf{x})$$

Rays are normals to surfaces of constant phase
If rays do not intersect we get an image (eikon)

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Solar concentrators are not seeking an image of the sun!

Nonimaging Optics

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Nonimaging optics

From Wikipedia, the free encyclopedia

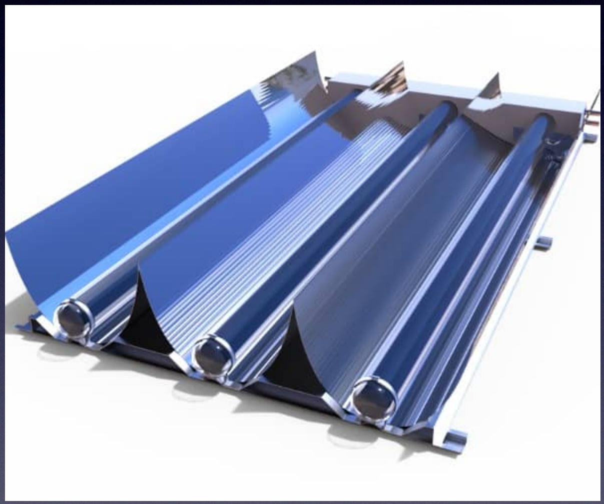
Nonimaging optics (also called **anidolic optics**)[1][2][3] is the branch of optics concerned with the optimal transfer of light radiation between a source and a target. Unlike traditional imaging optics, the techniques involved do not attempt to form an image of the source; instead an optimized optical system for optimal radiative transfer from a source to a target is desired.

Welford and Winston (1978)

- Seeks thermodynamically optimal designs
- Applications:
  - solar concentrators that do not track the sun
  - illumination engineering

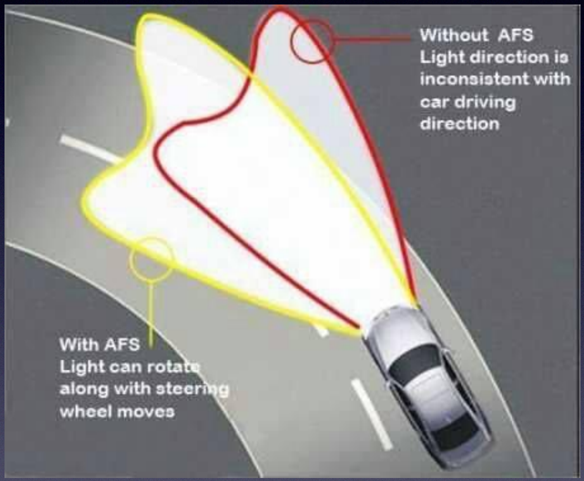
Nonimaging Concentrators

Faceted Winston cone [Winston Lab]



Faceted trough concentrator [Arctic Solar]

Nonimaging Lighting

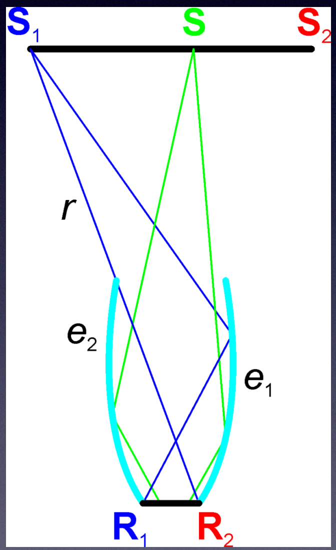


Toyota RAV4

Nonimaging Designs

Planar (2D) nonimaging concentrators

- Hottel strings, flow-lines [Winston *et al.* (2004)]
- Designs can achieve
  - All light inside a cone reaches the receiver
  - Thermodynamically ideal in 2D



Limitations of Design Methods

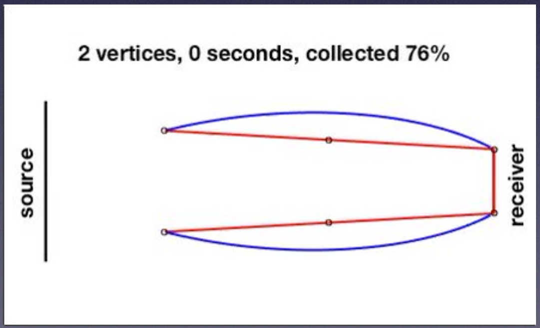
- 3D designs obtained by symmetry (cone / trough)
  - Not ideal
  - Optimal?
  - Fairly large

- Faceted and truncated designs are more practical
  - Not ideal
  - No analytical method

Computational optimization methods are needed

Computational Approaches

- Ray-tracing + *ad hoc* optimization
- SMS design method [Miñano & Benitez]
- Ray-tracing + Pattern Search [Ilan *et. al*]



Can benefit from new ideas

Nonimaging Theory

With Arnold Kim

What is the (differential) equation governing nonimaging optics?

Radiative Transfer Theory

Incoherent light can be described by its radiance

Chandrasekhar

It is natural to consider nonimaging optics in this context

Radiative Transport Equation (RTE)

$$\mathbf{s} \cdot \overrightarrow{\nabla} L \;\; + \;\; \mu_t L \;\; = \;\; \mu_s \int_{S^{d-1}} p(\mathbf{s}, \mathbf{s}') L(\mathbf{x}, \mathbf{s}')\, d\mathbf{s}' \; + \; Q(\mathbf{x}, \mathbf{s})$$

transport absorption scattering sources / sinks

Neglects phase-dependent phenomena

- diffraction
- interference

Radiative Transport Equation (RTE)

$$\mathbf{s} \cdot \overrightarrow{\nabla} L \quad + \quad \mu_t L \quad = \quad \mu_s \int_{S^{d-1}} p(\mathbf{s}, \mathbf{s}') L(\mathbf{x}, \mathbf{s}')\, d\mathbf{s}' \ + \ Q(\mathbf{x}, \mathbf{s})$$

transport absorption scattering sources / sinks

In free-space or homogeneous medium:

$$\mathbf{s} \cdot \overrightarrow{\nabla} L(\mathbf{x}, \mathbf{s}) = 0$$

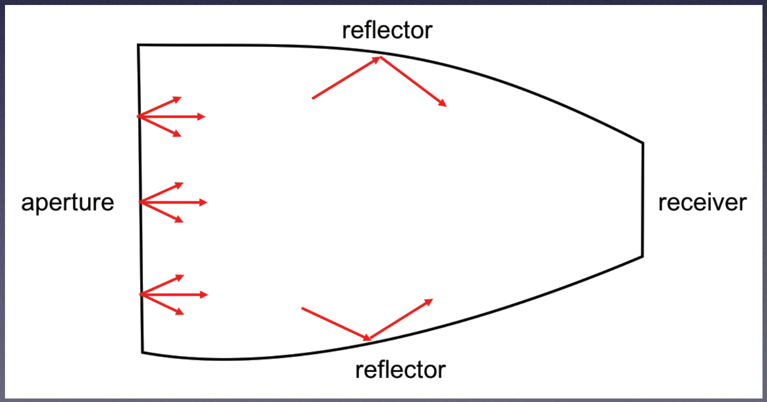
(linear transport eqn)

Governing equation of nonimaging optics

Boundary Conditions

$$\mathbf{s} \cdot \overrightarrow{\nabla} L = 0$$

Need to specify $L(\mathbf{x},\mathbf{s})$ in all directions entering the domain



- Effective source at the aperture
- Specular reflector

Two Models of Light

Coherent optics	*Radiation optics*
$\lvert \overrightarrow{\nabla} S(\mathbf{x}) \rvert = n(\mathbf{x})$	$\mathbf{s} \cdot \overrightarrow{\nabla} L(\mathbf{x}, \mathbf{s}) = 0$
phase	radiation
nonlinear 1^{st}-order PDE	linear 1^{st}-order PDE
rays	?

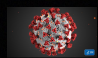
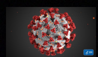
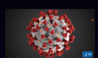
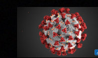
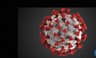
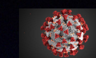
Problem Formulation

Need to solve

$$\mathbf{s} \cdot \overrightarrow{\nabla} L = 0 \quad + \ \textbf{boundary conditions}$$

Once *L* is found we can recover the light flux

Can we solve it?


Green's Function Formalism

Solution of a RTE BVP can be expressed as [Case (1969)]

$$L(\mathbf{x}, \mathbf{s}) = \int_{\partial D} \int_{S^{d-1}} \Gamma(\mathbf{x}, \mathbf{x}', \mathbf{s}, \mathbf{s}')\ (\mathbf{n}(\mathbf{x}') \cdot \mathbf{s}') L(\mathbf{x}', \mathbf{s}')\, d\mathbf{s}' d\mathbf{x}'$$

- Spatial integral only on $\partial D =$ {aperture, reflector, receiver}
- Γ is called the *fundamental solution*
- $\mathbf{n}(\mathbf{x}')$ is the normal to the boundary

Green's function reduces the 3D problem to a surface integral

Challenges

$$L(\mathbf{x},\mathbf{s}) = \int_{\partial D}\int_{S^{d-1}} \Gamma(\mathbf{x},\mathbf{x}',\mathbf{s},\mathbf{s}')\ (\mathbf{n}(\mathbf{x}')\cdot\mathbf{s}')L(\mathbf{x}',\mathbf{s}')\,d\mathbf{s}'d\mathbf{x}'$$

1. For RTE, Γ cannot be obtained explicitly
2. Angular integral is over all directions
    - We only know *L*(**x,s**) entering the aperture

⇒This is an integral equation for *L*(**x,s**)

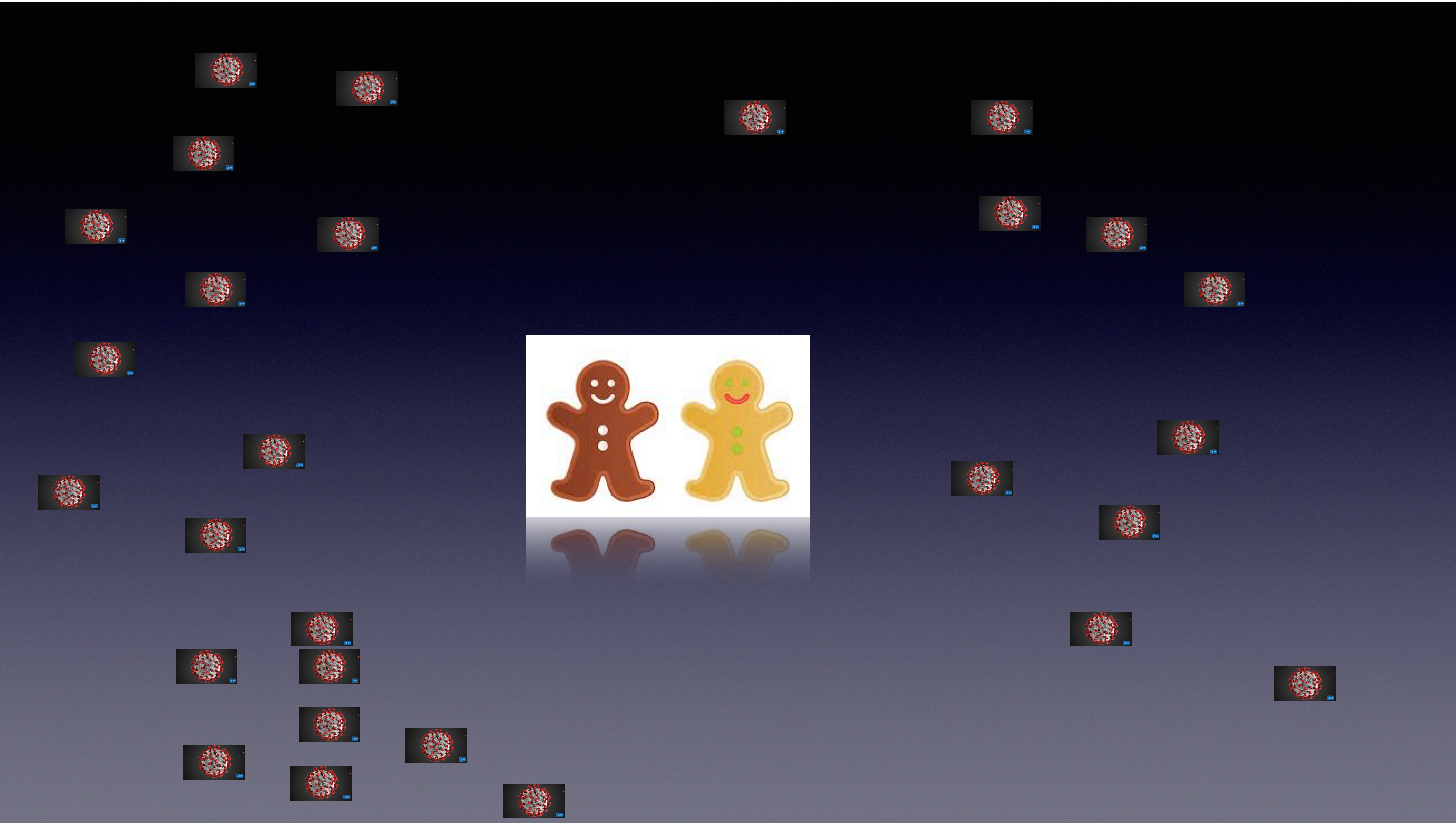
Fundamental Solution

"point charge"

$$\mathbf{s} \cdot \vec{\nabla} \Gamma(\mathbf{x}, \mathbf{x}', \mathbf{s}, \mathbf{s}') = \delta(\mathbf{x} - \mathbf{x}')\delta(\mathbf{s} - \mathbf{s}')$$

Kim and Ilan: $\Gamma(\mathbf{x}, \mathbf{x}', \mathbf{s}, \mathbf{s}') = \underbrace{\mathbb{1}_{R(\mathbf{x}',\mathbf{s})}(\mathbf{x})}_{\text{ray function}} \; \delta(\mathbf{s} - \mathbf{s}')$ *Explicit formula*

$$\mathbb{1}_{R(\mathbf{x}_0,\mathbf{s})}(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \in R(\mathbf{x}_0, \mathbf{s}) \\ 0 & \mathbf{x} \notin R(\mathbf{x}_0, \mathbf{s}) \end{cases} \qquad R(\mathbf{x}_0, \mathbf{s}) = \{\mathbf{x}_0 + t\mathbf{s} \mid \forall t \geq 0\}$$



Ray Functions

Translational symmetry $\mathbb{1}_{R(\mathbf{x}_0,\mathbf{s})}(\mathbf{x}) = \mathbb{1}_{R(\mathbf{0},\mathbf{s})}(\mathbf{x} - \mathbf{x}_0)$

Fourier transform $F[\mathbb{1}_{R(\mathbf{x}',\mathbf{s})}(\mathbf{x})](\mathbf{k}) = \dfrac{e^{-i\mathbf{k}\cdot\mathbf{x}'}}{i(\mathbf{s}\cdot\mathbf{k})} + \pi\delta(\mathbf{s}\cdot\mathbf{k})$

This generalizes the 1D formula

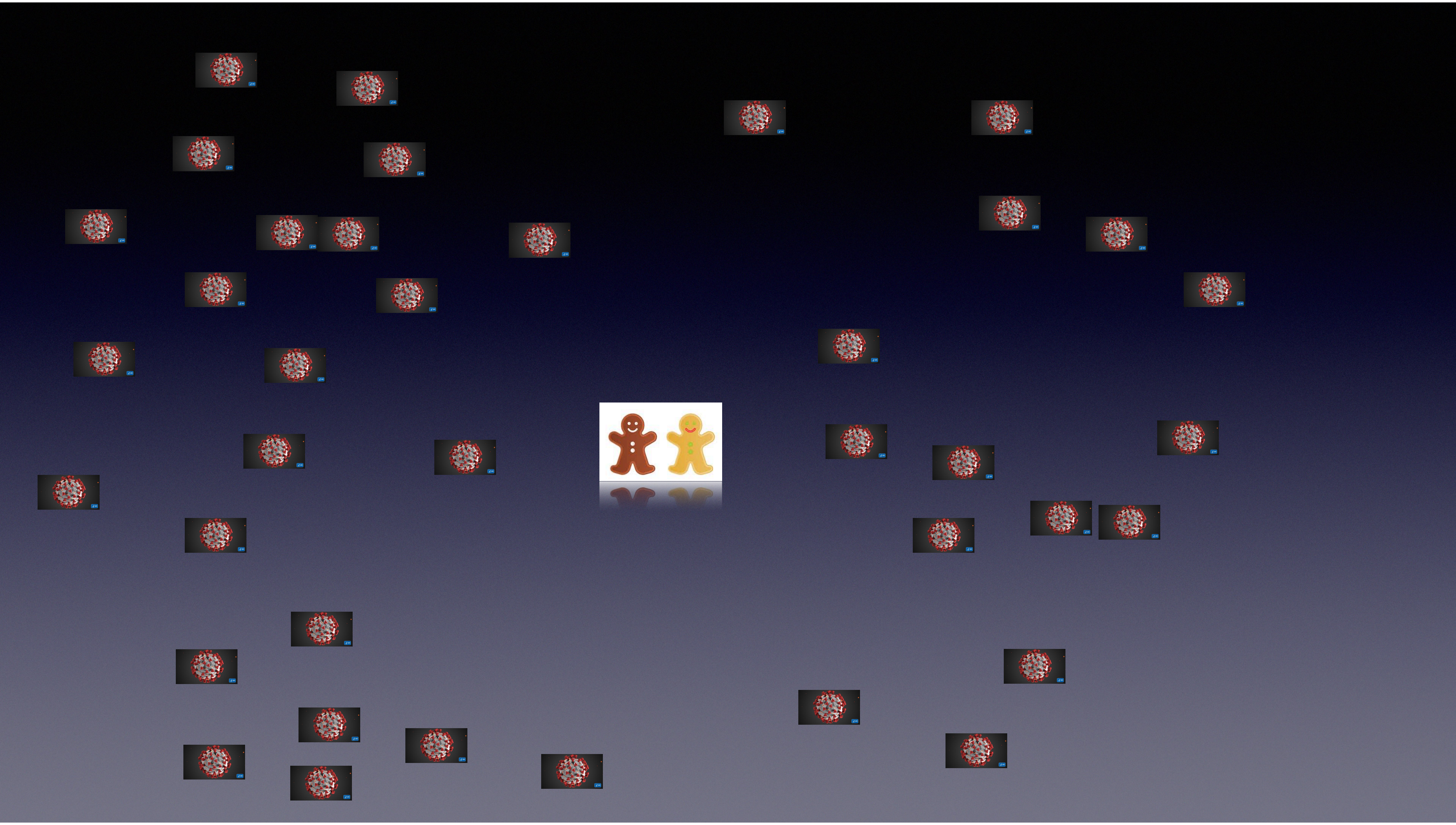
$$F[H(x)] = \frac{1}{ik} + \pi\delta(k)$$

Kim and Ilan [SPIE 2018]

Application to Nonimaging Optics

$$L(\mathbf{x},\mathbf{s}) = \int_{\partial D}\int_{S^{d-1}} \mathbb{1}_{R(\mathbf{x}',\mathbf{s})}(\mathbf{x})\ (\mathbf{n}(\mathbf{x}')\cdot\mathbf{s}')L(\mathbf{x}',\mathbf{s}')\,d\mathbf{s}'d\mathbf{x}'$$

- Reduces the problem to a surface integral equation
- Enables new approaches to compute the radiance
    - Further work is needed



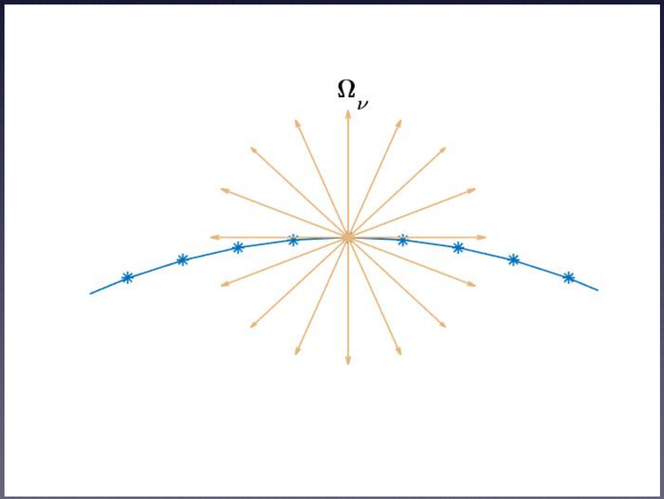
Computational Approach

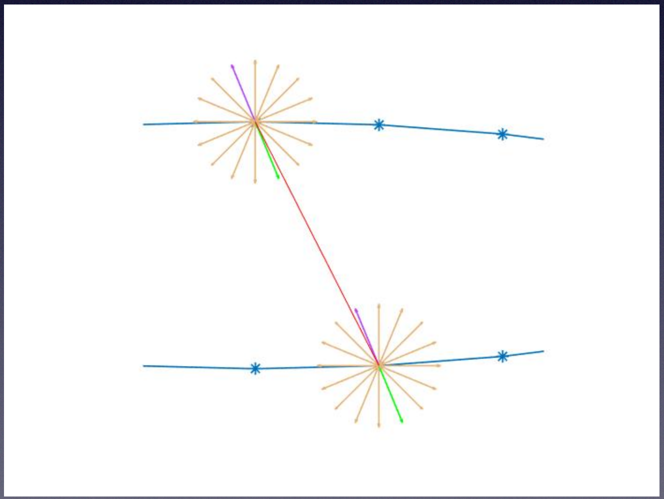
$$L(\mathbf{x},\mathbf{s}) = \int_{\partial D}\int_{S^{d-1}} \mathbb{1}_{R(\mathbf{x}',\mathbf{s})}(\mathbf{x})\ (\mathbf{n}(\mathbf{x}')\cdot\mathbf{s}')L(\mathbf{x}',\mathbf{s}')\,d\mathbf{s}'d\mathbf{x}'$$

In there a Boundary Integral Method formulation?

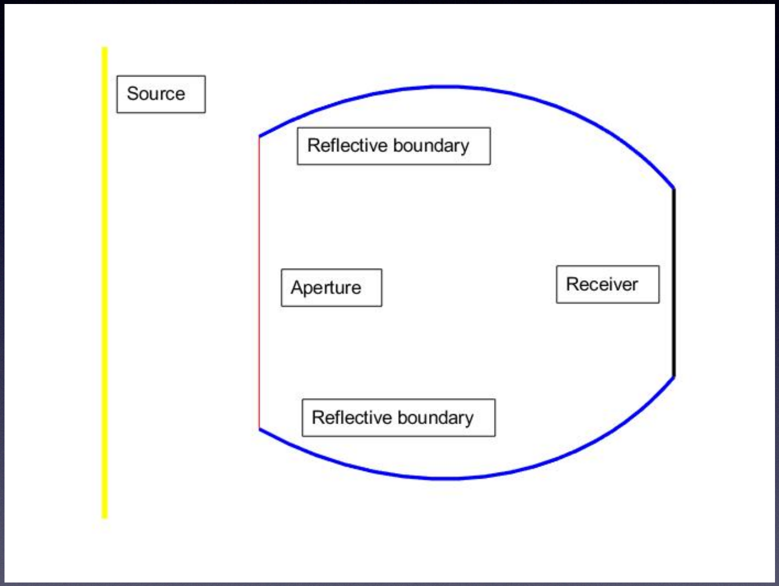
Ray Matching

With Pedro González-Rodríguez [In Progress]



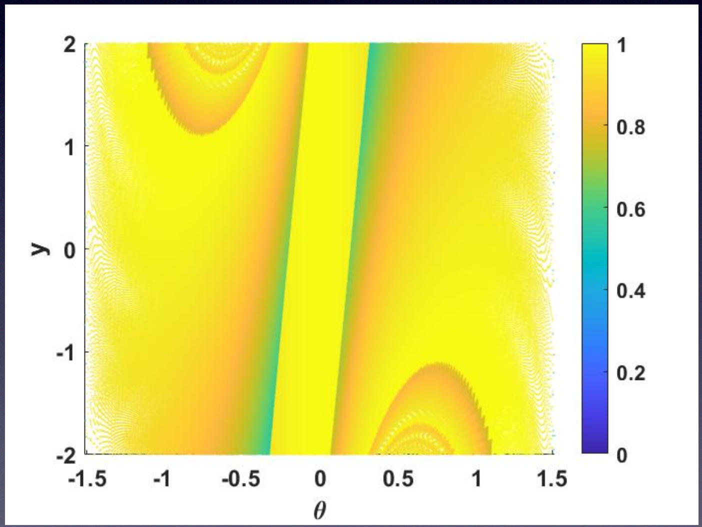


Test Problem

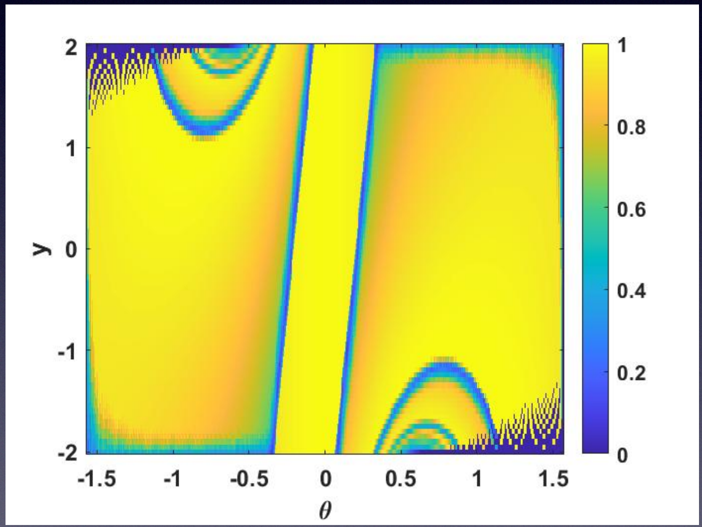


Compute $L(y, \theta)$ at the receiver

Ray Matching vs. Ray Tracing



Ray tracing



Ray matching

The Future

Conclusions

Radiative transfer theory provides a framework for nonimaging optics

Potential application to designing

- solar concentrators
- illumination systems
- computer graphics

Thank you for your attention!