UC Merced: MATH 21 — Exam #1 — 05 October 2005

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Sprague) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of seven problems on both sides of this paper and a total of 100 points. Please start each of the seven problems on a new page.

1. (20 points) Answer the following Always True (T), False (F), or Not Enough Information (NEI). Only your final answer will be graded on these problems.
   (a) If $f(0) = 5$ and $f'(0) = 1$ and we know that $f''(x) < 0$ for $x \geq 0$, then we know that $f(5) < 10$.
   (b) If a function is continuous on some interval it is also differentiable on that interval.
   (c) The function $\ln(x)$ is differentiable for all $x > 0$.
   (d) $\frac{\mathrm{d}}{\mathrm{d}v}\left(\sin(3\pi/2)v^2 + \pi\right) = -2v$.
   (e) If we know the value for $\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=5}$, then we know that $f(7) = f(5) + 2\left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=5}$.
2. (15 points) Given the piecewise function $f(x)$ defined below, find the value of b such that the function is continuous at $x = 1$. Once you've found an appropriate value for b, sketch the function over the interval $[-1, 4]$.
$$f(x) = \begin{cases} e^{-(x-1)} & \text{for } x \leq 1 \\ -(x-3)^2 + b & \text{for } x > 1 \end{cases}$$
3. (12 points) Find the requested limits, if they exist. If they do not exist, explain.
   (a) $\lim_{x \to 4} \dfrac{\ln[\cos(\pi x/4)]}{x - 2}$
   (b) $\lim_{x \to 2} \sin(x-2)\dfrac{|x-2|}{x-2}$
4. (25 points total) Find the requested information in the following unrelated problems:
   (a) (10 points) Using the definition of a derivative, calculate $g'(x)$ where $g(x) = \sqrt{x+1}$.
   (b) (10 points) Find the equation of the line tangent to $m(x) = \sqrt{x} + 5x^2$ at the point $x = 4$.
   (c) (5 points) If $k(area)$ is a function representing the number of hours required for Mr. Johnson, a painter, to paint an $area$ (in m^2) of a house, what are the units of $k''(area)$? If Mr. Johnson always paints more slowly as the day goes on, what do you expect the sign of $k''(20)$ to be?

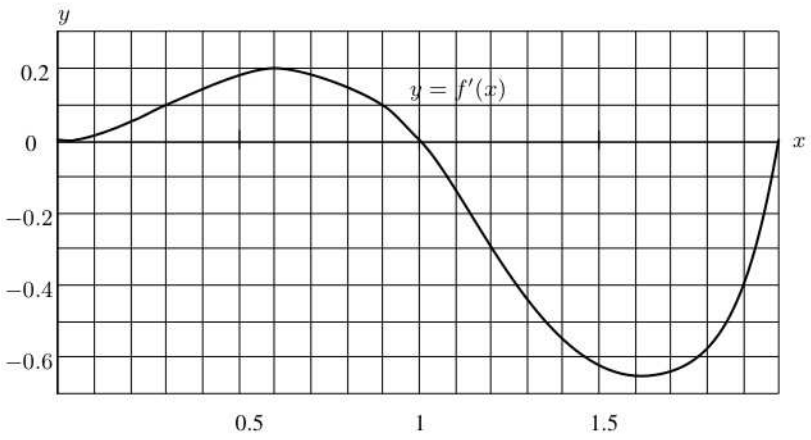
THERE'S MORE ON THE BACK!

5. (12 points) Consider the graph shown below for $y = f'(x)$, and answer the following three questions:

   (a) On what intervals is f increasing? On what intervals is it decreasing?

   (b) On what intervals is the graph of f concave up? Concave down?

   (c) Which value is greater: $f(0.25)$ or $f(1)$?



6. (8 points) Consider the data shown in the table below for the height of an object $y(t)$ as a function of time t. Answer the following two questions:

   (a) What is the average velocity for $2 \leq t \leq 5$ sec?

   (b) What is your best estimate for the instantaneous velocity at $t = 3$ sec?

t (sec)	0	1	2	3	4	5	6
y (feet)	10	45	70	85	90	85	70

7. (8 points) Consider the graph of $f(x)$ shown below; the domain of $f(x)$ is $-2 \leq x \leq 2$. Make a sketch of $f'(x)$, being sure to label any numerical values of $f'(x)$ that you know.