UC Merced: MATH 21 — Exam #2 — 02 November 2005

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Sprague) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of six problems on both sides of this paper and a total of 100 points. Please start each of the six problems on a new page.

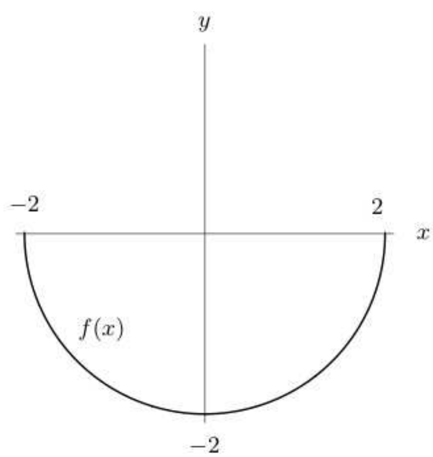
1. (20 points) Answer the following Always True (T) or False (F). Only your final answer will be graded on these problems.
   (a) If $f(x) = \frac{\cos(\pi x)}{1-x}$, the Mean Value Theorem tells us that, at some point c in $[0, 2]$, $f'(c) = -1/2$.
   (b) The domain of $\arctan(x)$ is $(-\infty, \infty)$
   (c) $\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\sinh(2\theta^3) - \cosh(2\theta^3)\right] = 0$
   (d) $\cos\left[\arcsin(x^3)\right] = \sqrt{1 - x^3}$
   (e) If $f'(x) = g'(x)$, for all x, then $f(x) = g(x)$ for all x.

2. (12 points) Using the definition of the derivative, show that $\frac{\mathrm{d}}{\mathrm{d}x}\sin(x) = \cos(x)$, given that

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1 \quad \text{and} \quad \lim_{\theta \to 0} \frac{\cos(\theta) - 1}{\theta} = 0$$

3. (28 points total) Find the requested information in the following unrelated problems:
   (a) Find $h'(t)$ for $h(t) = t^{\pi^3} + (\pi^3)^t + \pi t$
   (b) Evaluate $\frac{\mathrm{d}}{\mathrm{d}\theta}\ln[\theta\cos(\theta)]$
   (c) Evaluate $\frac{\mathrm{d}}{\mathrm{d}x}e^{[5^x + x\ln(x)]}$
   (d) Calculate the slope of the line perpendicular to the curve defined by the equation $y = \frac{x}{y+a}$ at the point $(0, 0)$ for some constant a.

4. (8 points) Consider the graph of $f(x)$ shown below; the domain of $f(x)$ is $-2 \leq x \leq 2$. Make a sketch of $f'(x)$, being sure to label any numerical values of $f'(x)$ that you know.



THERE'S MORE ON THE BACK!

5. (24 points) Consider $g(x)$, which is defined on $-\infty < x < 1.5$ as follows:

$$g(x) = \begin{cases} x^2 - x\,, & -\infty < x \le 1 \\ x\ln(x) - x + 1\,, & 1 < x < 1.5 \end{cases}$$

$$\frac{\mathrm{d}g}{\mathrm{d}x} = \begin{cases} 2x - 1\,, & -\infty < x < 1 \\ \ln(x)\,, & 1 < x < 1.5 \end{cases}$$

$$\frac{\mathrm{d}^2 g}{\mathrm{d}x^2} = \begin{cases} 2\,, & -\infty < x < 1 \\ \frac{1}{x}\,, & 1 < x < 1.5 \end{cases}$$

(a) Find all critical points of $g(x)$. On what interval(s) is the function increasing? decreasing? Identify any local maximum or minimum points.

(b) On what interval(s) is the function concave up? concave down? Identify any inflection points.

(c) Make a sketch of $g(x)$ on the interval $-1 \le x < 1.5$. Be sure to label the graph and be as accurate as possible.

(d) Identify any global maximum or minimum points.

6. (8 points) Consider, again, $g(x)$ from the previous problem.

(a) Find the tangent line approximation to $g(x)$ at $x = -1$.

(b) Calculate the error of the approximation at $x = 1.2$.