# UC Merced: MATH 21 — Exam #3 — 30 November 2005

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Sprague) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There are a total of five problems and a total of 100 points. Please start each of the five problems on a new page.

1. **(20 points)** Answer the following Always True (T) or False (F). Only your final answer will be graded on these problems.

    (a) $\int_a^b f(x)dx$ may be described as the area between the function $f(x)$ and the $x$-axis over the interval $a \leq x \leq b$.

    (b) $\int \cos(x/5)dx = 5\sin(x/5) + 5$

    (c) When marginal cost is equal to marginal revenue, profit is maximized.

    (d) A set of parametric equations describing motion along the line $y = 3x + 4$ is given by $y = 7 + 3t$ and $x = 1 + t$.

    (e) All continuous functions are integrable.

2. **(10 points)** Using the definition of the derivative, show that $\frac{d}{dx}\sin(x) = \cos(x)$, given that

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1 \quad \text{and} \quad \lim_{\theta \to 0} \frac{\cos(\theta) - 1}{\theta} = 0$$

3. (30 points total) Find the requested information in the following unrelated problems:

    (a) **(8 points)** A particle is located $x = 1$ at $t = 0$, and its velocity is given by $v(t) = t^2$. Use left-endpoint sum and $\Delta t = 1/2$ to estimate the position of the particle at $t = 3/2$. Is your approximation an over or under estimate for the true position?

    (b) **(6 points)** Evaluate $\int \frac{(x+1)}{x^2} dx$

    (c) **(8 points)** Evaluate the following limit:

$$\lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{\sin(x)} \right)$$

    (d) **(8 points)** Given the plot of $g(x)$ given below, calculate $\int_0^4 g(x)\, dx$
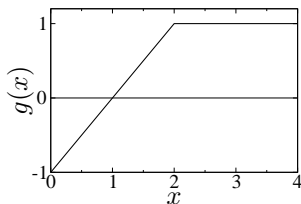


4. **(20 points)** We wish to design a closed cylindrical container with a volume of 1 m³. It costs twice as much for the material used for the lids than that used for the sidewalls. What is the cylinder radius that will minimize the material cost.

5. **(20 points)** I was driving to work along Hwy 140 towards the intersection with Arboleda Drive, along which a police car was approaching Hwy 140. Arboleda intersects Hwy 140 at a right angle. The police car was approaching the intersection at a constant rate of 50 mph. When the police officer was one mile from the intersection, and I was three miles from the intersection, his laser indicated that the distance between us was decreasing at a rate of 70 mph. How fast was I driving?