UC Merced: MATH 21 - Exam #2 - 23 March 2006

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Bianchi), (4) your section number, and (5) a grading table (see front board). Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators, and crib sheets are not permitted. There are a total of five problems on both sides of this paper and a total of 100 points. Please start each of the five problems on a new page.

1. (20 Points) Answer the following Always True (T) or False (F). Only your final answer will be graded on these problems.
   (a) If $f'(x) = g'(x)$, then $f(x) = g(x)$.
   (b) If $a < b$ and f' is positive on $[a, b]$ then $f(a) < f(b)$.
   (c) $\frac{d}{dx}\left[\cosh(x^2)\right] = 2x\sinh(x^2)$.
   (d) Suppose $f(x)$ and $g(x)$ are continuous and that $f'(x), f''(x), g'(x), andg''(x)$ exist for all values of x. If $y = f(x)g(x)$, then $\frac{d^2y}{dx^2} = f''(x)g(x) + f(x)g''(x)$.
   (e) If f is defined for all x, and f has a maximum at $x = 1$, then $f'(1) = 0$.

2. Solve the following unrelated problems.
   (a) (10 points) Revenue is given by $R(q) = 50q - q^2$ and the total cost is $C(q) = 100 + 2q$ where q is the number of items made and sold. How many items should you make and sell to maximize profit? How do you know that this value corresponds to maximum profit?
   (b) (10 points) Find the linear approximation to the function $y = \dfrac{3x + ln(x)}{x^2}$ near $x = 1$. Use this to approximate y where $x = \frac{5}{4}$.
   (c) (10 points) Find $\frac{dy}{dx}$ if $y = \cos(\arcsin(x^2))$.

3. Suppose that $f(x) = x^3 - 3x + 2$.
   (a) (10 points) Find and classify all local extrema. On what interval(s) is f increasing? Decreasing?
   (b) (10 points) Find any inflection points. On what interval(s) is f concave up? Concave down?

4. (18 points) You own property on a river. You want to fence in a rectangular piece of land next to the river, so that you only have to fence in 3 sides (the fourth side is the river's edge). You have 1000 feet of fence. What dimensions should you make your fenced piece of land in order to enclose the largest possible area? How do you know that this value corresponds to a maximum area?

5. (12 points) Draw a possible graph of $y = f(x)$ if $f(x)$ has all of the following properties. (You should have one function f that has all of these properties.)
   - $f(x)$ is defined on the interval $[-2, 4]$.
   - $f'(x) > 0$ for $x < 0$.
   - $f'(x) = 0$ for $0 < x < 1$.
   - $f'(x) < 0$ for $1 < x$.
   - $f'(x)$ is undefined at $x = 0$ and $x = 1$.
   - $f''(x) > 0$ for $x < 0$.
   - $f''(x) < 0$ for $x > 1$.